effectiveness

uniformity

[GenEll]

[Belyi] non-critical Belyi map

Vojta general case

Vojta special case

concretely computable

cpt'ly bdd domain

[IUTch I ~ IV]

$d^{4+\epsilon}$

# §10. Hodge-Arakelov-theoretic Evaluation

[IUTch II, Rem 1.4.1]

## §10.1 Radial Environment

**Def 10.1** [IUTch II, Ex 1.7]

$(\mathcal{R}, \mathcal{C}, \Phi)$ : radial environment

$\overset{\text{def}}{\iff}$ $\mathcal{R}, \mathcal{C}$ : cat's s.t. $\forall$ hom's are isom's

$\Phi : \mathcal{R} \to \mathcal{C}$ (alg'/ori'ally def'd) functor s.t. ess. surj.

## §10.1 Radial Environment

**Def 10.1** [IUTch II, Ex 1.7]

(1). $(\mathcal{R}, \mathcal{C}, \Phi)$ : radial environment

$\overset{\text{def}}{\Longleftrightarrow}$ $\mathcal{R}, \mathcal{C}$ : cat's s.t. $\forall$ hom's are isom's

$\Phi : \mathcal{R} \longrightarrow \mathcal{C}$ (alg'orithmically def'd) functor s.t. ess. surj.

$\mathcal{R}$ : radial data

$\mathcal{C}$ : coric data

$\Phi$ : radial alg'm

(2). $(\mathcal{R}, \mathcal{C}, \Phi)$ : multiradial env.

$\overset{\text{def}}{\Longleftrightarrow}$ $\Phi$ : full

$(\mathcal{R}, \mathcal{C}, \Phi)$ : uniradial

$\overset{\text{def}}{\Longleftrightarrow}$ $(\mathcal{R}, \mathcal{C}, \Phi)$ : not multiradial

(3). $(\mathcal{R}, \mathcal{C}, \Phi)$, $({}^{\dagger}\mathcal{R}, {}^{\dagger}\mathcal{C}, {}^{\dagger}\Phi)$ : rad. envs

$$\begin{array}{ccc} \mathcal{R} & \xrightarrow[\Phi_{\mathcal{R}}]{} & {}^{\dagger}\mathcal{R} \\ \Phi \downarrow & \circlearrowright & \downarrow {}^{\dagger}\Phi \\ \mathcal{C} & \xrightarrow[\Phi_{\mathcal{C}}]{} & {}^{\dagger}\mathcal{C} \end{array}$$

given $\Phi_{\mathcal{R}}$ : multiradially defined

$\overset{\text{def}}{\Longleftrightarrow}$ $(\mathcal{R}, \mathcal{C}, \Phi)$ : mult. env.

e.g. (but $\Phi$ ...)

radial data

$\mathcal{R}$ Obj $(\Pi, G, \alpha)$

$\exists\, \Pi \cong \Pi^{\text{tp}}_{\underline{X}_v}$, $G \cong G_v$, $\alpha : \Pi/\Delta \xrightarrow{\sim} G$ full poly

$\Pi \rightsquigarrow \Delta$

Hom $(\Pi, G, \alpha) \xrightarrow{\sim} (\Pi^{*}, G^{*}, \alpha^{*})$

pair of isom's $\left( \begin{array}{l} \Pi \xrightarrow{\sim} \Pi^{*} \\ G \xrightarrow{\sim} G^{*} \end{array} \right.$

$\cong G$

$\cong G_v$

$\cong G$

## Conic data

$\mathcal{C}$ Obj $G$

$\exists \simeq G_k$

Thm $G \cong G^{\vee}$ isom.

$\Phi(\Pi, G, \alpha) := G$

$\rightsquigarrow \Phi$: full $\rightsquigarrow (\mathcal{R}, \mathcal{C}, \Phi)$: multirad. env.

$^{\dagger}\Pi$ h.d.

mono-an $^{\circ}G$ — $^{\ddagger}\Pi$ h.d.

cf. $\Pi \overset{\Phi}{\longmapsto} \Pi/\Delta$ not full $\rightsquigarrow$ unirad.

cf. $\mathrm{Aut}(\bar{k}/k) \underset{\neq}{\hookrightarrow} \mathrm{Out}(G_k)$ $(p > 2)$

$\mathcal{C}$ h.d. $\mathcal{R}$ obj $(C, V, \alpha)$ full poly

real an $\mathbb{R}^2$ $\xleftarrow{\Phi}$ $\mathcal{C}$ h.d.

$G \overset{\exists}{\simeq} G_k \rightsquigarrow \mathcal{O}^{\times\mu}(G)$

$\mathrm{Ismet}(G) := \{$ $G$-isometries of $\mathcal{O}^{\times\mu}(G)$, i.e. $\alpha: \mathcal{O}^{\times\mu}(G) \xrightarrow{\sim} \mathcal{O}^{\times\mu}(G)$ $G$-equiv. s.t. $G \supset^{\forall} H$ open preserves $\mathrm{Im}\{\mathcal{O}^{\times}(G)^{H} \to \mathcal{O}^{\times\mu}(G)^{H}\}$ integral str. $\}$

cpt. top. [illegible]

preserved by $G_1 \simeq G_2$

$G \mapsto \mathrm{Ismet}(G)$

$\mathrm{Ismet}(-) \rightsquigarrow \mathrm{Ismet}$

$\mathrm{Im}(\hat{\mathbb{Z}}^{\times}) = \mathrm{Im}\{\hat{\mathbb{Z}}^{\times} \to \hat{\mathbb{Z}}_{p}^{\times} \hookrightarrow \mathrm{Ismet}(G)\}$

$\mathrm{Im}(\mathbb{Q}_p^{\times}, \mathbb{Z}_p^{\times})$

$\mathrm{Im}(\mathbb{Q}_p^{\times}, \mathbb{Z}_p^{\times})$

(cf. [IUTchI, Rem. 1.11.1] $1 \to \mathrm{Im}(\hat{\mathbb{Z}}^{\times}) \to \mathrm{Aut}(G \curvearrowright \mathcal{O}^{\times\mu}(G)) \to \mathrm{Aut}(G) \to 1$)

cf. $1 \to \mathrm{Ismet}(G) \to \mathrm{Aut}(G \curvearrowright \mathcal{O}^{\times\mu}(G)) \to \mathrm{Aut}(G) \to 1$ Ism

$^{\circ}\mathcal{F}$ unit "family" $\mathfrak{S}$ (mono-an. / [illegible])

$\mathcal{F}$ value "one piece" $\mathfrak{D}$

$^{\circ}\mathcal{F}$'d Aut., Ind

$\mathcal{P}$ $\mathcal{P}^{\natural}$

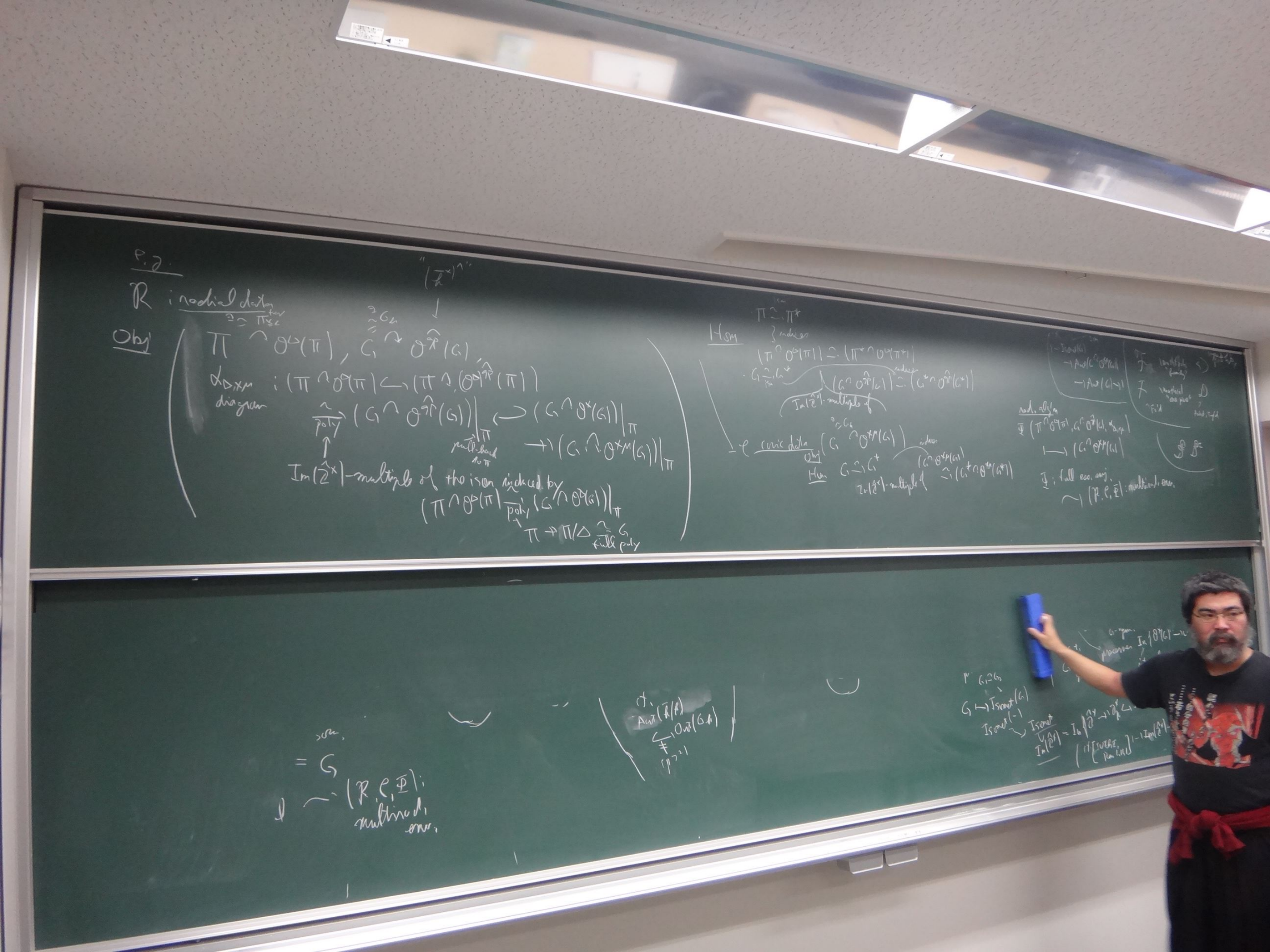
"$(\bar{k}^{\times})^{\wedge}$"

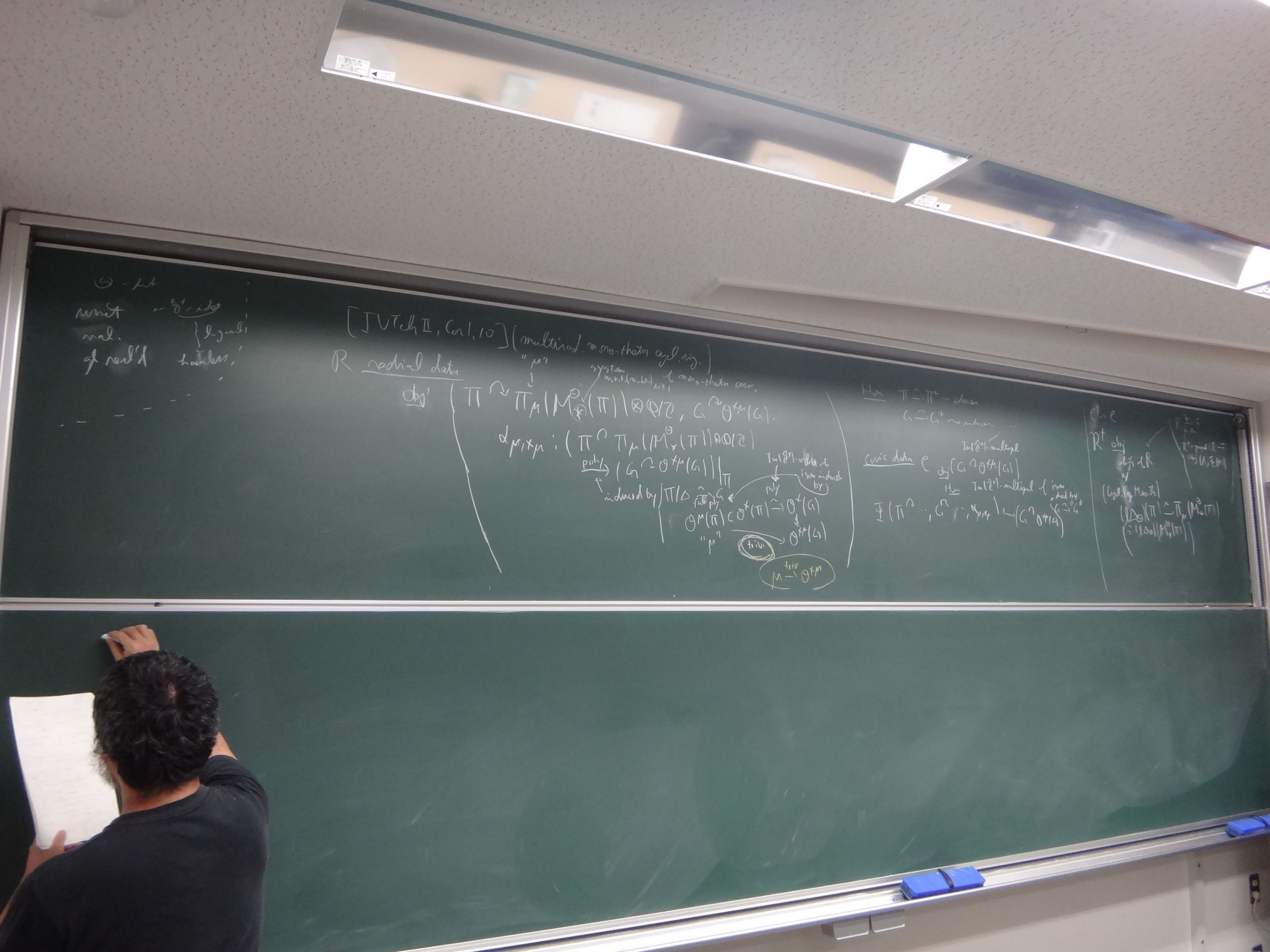
$\downarrow$

e.g.

$\mathcal{R}$: radial data for $\Pi \overset{\exists}{\simeq} \Pi_{X_k}$

$(\Pi \curvearrowright \mathcal{O}^{\triangleright}(\Pi),\ G \overset{\exists}{\simeq} G_k \curvearrowright \mathcal{O}^{\hat{\times}}(G),$


e.g.
R : radial data
Obj
Hom
diagram
Im(Ẑ^×)-multiple of the isom induced by
full poly
coric data
Φ : full ess. surj.
multirad. env.
= G


[IUTch II, Cor 1.10] (multirad. mono-theta cycl. rig.)
R radial data
obj'
system
mono-theta env.
induced by
poly
triv
triv

$\overset{\dagger}{\mathcal{R}} \xrightarrow{\text{functor}} \mathcal{R}^{\dagger}$

$(\Pi^{\curvearrowright} \cdots, G^{\curvearrowright} \cdots, \alpha_{\mu, \times\mu})$

$\mapsto (\Pi^{\curvearrowright} \cdots, G^{\curvearrowright} \cdots, \alpha_{\mu, \times\mu}, (\text{Cycl. Rig. Mono-Th}))$

multiradially defined

$$\begin{array}{ccc} \mathcal{R} & \longrightarrow & \mathcal{R}^{\dagger} \\ {\scriptstyle \Phi} \downarrow & \swarrow & \\ \mathcal{C} & & \end{array}$$

${}^{\dagger}\Pi$ ${}^{\dagger}\Pi_{\mu} \cong$ cycl. rig.

${}^{\ddagger}\Pi$ ${}^{\ddagger}\Pi_{\mu} \cong$ cycl. rig.

$G \curvearrowright \mathcal{O}^{\times\mu}(G) \supseteq \mathrm{Im}(\hat{\mathbb{Z}}^{\times})$

$\Pi_{\mu}(M^{\Theta}_{*}(\Pi)) \otimes \mathbb{Q}/\mathbb{Z}$

$\downarrow$ trivial

$\mathcal{O}^{\times\mu}(G)$

any rigidification on $\Pi_{\mu}(M^{\Theta}_{*}(\Pi))$ has no rel'n w/ the coricity of $\mathcal{O}^{\times\mu}(G)$ / the $\mathrm{Im}(\hat{\mathbb{Z}}^{\times})$-indet of $\mathcal{O}^{\times\mu}(G)$

(Cycl. Rig. LCFT) ← uses $\mathcal{O}^{\triangleright}$ → cannot be treated multiradially

(Cycl. Rig. Mono-Th) ← only uses $\mu$ → multirad.

$\Theta$-link

unit ← $\hat{\mathcal{O}}^{\times}$-indet

val. — log-vol

of real'd — hodders

[IUTch II, Cor 1.10] (multirad. mono-theta cycl. rig.)

$\mathcal{R}$ <u>radial data</u> <u>obj</u>

$(\Pi \curvearrowright \Pi_{\mu}(M^{\Theta}_{\circledast}(\Pi)) \otimes \mathbb{Q}/\mathbb{Z}, G \curvearrowright \mathcal{O}^{\times\mu}(G)$,

"$\mu$" — system w.r.t. $(\text{mod } N)_{N \geq 1}$ of mono-theta env.

$\alpha_{\mu, \times\mu} : (\Pi \curvearrowright \Pi_{\mu}(M^{\Theta}_{*}(\Pi)) \otimes \mathbb{Q}/\mathbb{Z})|_{\Pi} \xrightarrow{\text{poly}} (G \curvearrowright \mathcal{O}^{\times\mu}(G))|_{\Pi}$

induced by $\Pi/\Delta \xrightarrow[\text{full poly}]{\sim} G$

$\mathcal{O}^{\mu}(\Pi) \subset \mathcal{O}^{\times}(\Pi) \xrightarrow[\text{poly}]{\sim} \mathcal{O}^{\times}(G)$

$\mathrm{Im}(\hat{\mathbb{Z}}^{\times})$-orbit of isom induced by

"$\mu$" $\mathcal{O}^{\mu}(\Pi) \xrightarrow{\text{trivial}} \mathcal{O}^{\times\mu}(G)$

$\mu \xrightarrow{\text{triv}} \mathcal{O}^{\times\mu}$

<u>Hom</u> $\Pi \xrightarrow{\sim} \Pi^{\dagger}$ ~ induces ...

$G \xrightarrow{\sim} G^{\dagger}$ ~ induces ...

$\mathrm{Im}(\hat{\mathbb{Z}}^{\times})$-multiple

<u>coric data</u> $\mathcal{C}$ <u>obj</u> $(G \curvearrowright \mathcal{O}^{\times\mu}(G))$

<u>Hom</u> $\mathrm{Im}(\hat{\mathbb{Z}}^{\times})$-multiple of isom induced by $G \xrightarrow{\sim} G^{\dagger}$

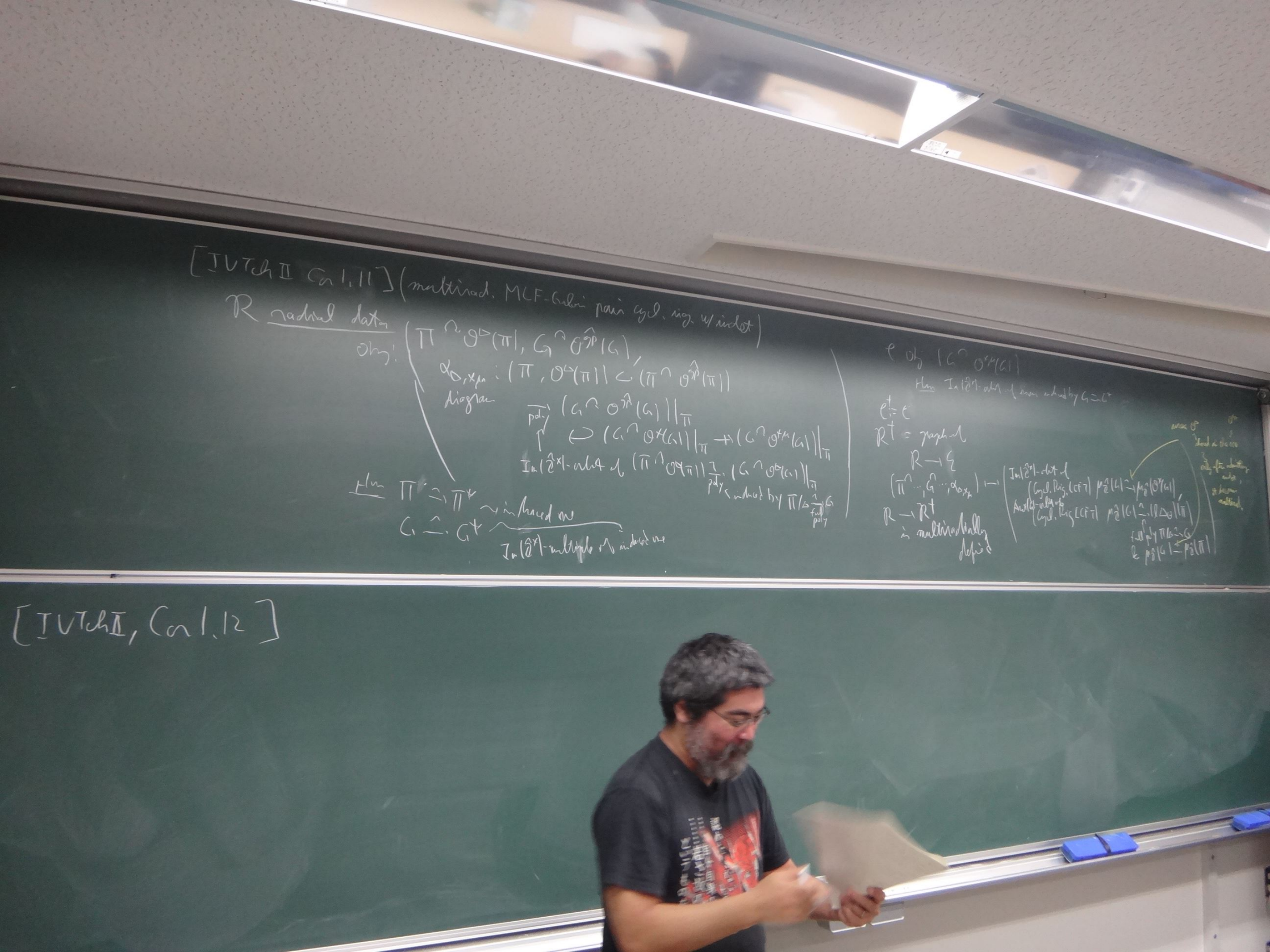
$\Phi(\Pi^{\curvearrowright} \cdots, G^{\curvearrowright} \cdots, \alpha_{\mu, \times\mu}) \mapsto (G \curvearrowright \mathcal{O}^{\times\mu}(G))$

$\mathcal{C}^{\dagger} := \mathcal{C}$

$\mathcal{R}^{\dagger}$ <u>obj</u> obj of $\mathcal{R}$ w/

(Cycl. Rig. Mono-Th) $(l\Delta_{\Theta})(\Pi) \xrightarrow{\sim} \Pi_{\mu}(M^{\Theta}_{*}(\Pi))$ $(\cong (l\Delta_{\Theta})(M^{\Theta}_{*}(\Pi)))$

$\mathcal{R} \xrightarrow[\text{functor}]{\Sigma\text{-}\ddagger}$ $\mathcal{R}^{\dagger}$ = groupoid $(\mathcal{R}$ — obj $(A, \Sigma(A)))$



[IUTchII Cor 1.11]
R radial data
Obj:
diagram
Thm
R → R†
in multiradially defined
[IUTchII, Cor 1.12]

[IUTchII, Cor 1.12] (multiradial constant mult. rig.) : const. mult. rig. ~ splitting; pilot ~ log-Kummer; radial gr. ... non-interference

$\mathcal{R}$ : radial data, Obj: $\left(\Pi \curvearrowright \Pi_{\mu}(\mathcal{M}^{\Theta}_{*}(\Pi)) \otimes \mathbb{Q}/\mathbb{Z},\ G \curvearrowright \mathcal{O}^{\times\mu}(G),\ \alpha_{\mu,\times\mu}\right)$ "$\mu$"

Hom: isom. induced by $\Pi \cong \Pi^{*}$

$\Phi$

$\mathcal{C}$ : coric data, Obj: $\left(G \curvearrowright \mathcal{O}^{\times\mu}(G)\right)$

Hom: $\mathrm{Ism}(\hat{\mathbb{Z}}^{\times})$-multiple of isom. induced by $G \cong G^{*}$

$\left(G \curvearrowright \mathcal{O}^{\times\mu}(G)\right)$

$\mathcal{C}^{\dagger} := \mathcal{C}$

consider the following fct. alg'ms:

$\Pi \longmapsto \text{diagram } (\dagger_{\times\Theta})(\Pi)$ (theta fct.)

$$\begin{array}{lcccccc}
``\Pi" & \mathcal{O}^{\times}(\Pi) & \hookrightarrow & \mathcal{O}^{\times}(\Pi)\cdot\underline{\underline{\Theta}}_{\infty}(\Pi) & \hookrightarrow & \varinjlim_{J \subset \Pi \text{ open}} & H^{1}(\Pi_{\underline{\ddot{Y}}}(\Pi)|_{J}, (l\Delta_{\Theta})(\Pi)) \\
 & \downarrow\wr & & \downarrow\wr & & & \downarrow\wr \text{ cycl. rig. (Cor 1.10)} \\
``\mathcal{M}" & \underbrace{\mathcal{O}^{\times}(\mathcal{M}^{\Theta}_{*}(\Pi))}_{\mathrm{Aut}} & \hookrightarrow & \mathcal{O}^{\times}(\mathcal{M}^{\Theta}_{*}(\Pi))\cdot\underline{\underline{\Theta}}_{\infty,\mathrm{env}}(\mathcal{M}^{\Theta}_{*}(\Pi)) & \hookrightarrow & \varinjlim_{J \subset \Pi} & H^{1}(\Pi_{\underline{\ddot{Y}}}(\mathcal{M}^{\Theta}_{*}(\Pi))|_{J}, \Pi_{\mu}(\mathcal{M}^{\Theta}_{*}(\Pi)))
\end{array}$$

$=: \mathcal{O}^{\times}_{\infty}\cdot\underline{\underline{\Theta}}(\Pi)$

$\| \quad \mathcal{O}^{\times}_{\infty}\cdot\underline{\underline{\Theta}}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*}(\Pi))$

i) $\exists$ fct. gp th'c alg'm

$\Pi \longmapsto \{(\iota, D)\}(\Pi)$

$\iota$ : pointed inversion autom.

$\Delta_{\underline{\ddot{Y}}}(\Pi) := \Delta \cap \Pi_{\underline{\ddot{Y}}}(\Pi)$ - outer autom. of $\Pi_{\underline{\ddot{Y}}}(\Pi)$

"$\pm\sqrt{-1}$"

$\rightsquigarrow D \subset \Pi_{\underline{\ddot{Y}}}(\Pi)$ : $\Delta_{\underline{\ddot{Y}}}(\Pi)$-conj. class of ... ... ps

ii) $(\iota, D)$ : pt'd inversion autom.

restr. to $D \,(\subset \Pi_{\underline{\ddot{Y}}}(\Pi))$

$\rightsquigarrow$ comm. diag.

$\{\mathcal{O}^{\times}\cdot{}_{\infty}\underline{\underline{\Theta}}(\Pi) \ldots\}$

$\downarrow\wr$

$\{\mathcal{O}^{\times}\cdot{}_{\infty}\underline{\underline{\Theta}}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*}(\ldots\}$

i) ∃ fct. gp thic alg'm

$\Pi \longmapsto \{(\iota, D)\}(\Pi)$

pointed inversion autom.

"$\pm\sqrt{-1}$" $\Delta^{\ddot{y}}(\Pi) := \Delta \cap \Pi^{\ddot{y}}(\Pi)$ - outer autom. of $\Pi^{\ddot{y}}(\Pi)$

$\rightsquigarrow D \subset \Pi^{\ddot{y}}(\Pi)$ : $\Delta^{\ddot{y}}(\Pi)$-conj. class of closed subgps

ii) $(\iota, D)$ : pt'd inversion autom.

restr. to $D \,|\subset \Pi^{\ddot{y}}(\Pi))$

$\rightsquigarrow$ comm. diag

$\iota$-inv. part

$$\{\mathcal{O}^{\times}_{\infty}\theta(\Pi)\}^{\iota} \xrightarrow{\text{restr. to } D} \mathcal{O}^{\times}(\Pi) \;\left(\subset \varinjlim_{J\subset\Pi \text{ open}} H^1(J, (l\Delta_\Theta)(\Pi))\right)$$

$$\wr\| \qquad\qquad \wr\|$$

$$\{\mathcal{O}^{\times}_{\infty}\theta_{env}(M^{\theta}_{*}(\Pi))\}^{\iota} \xrightarrow{\text{restr. to } D} \mathcal{O}^{\times}(M^{\theta}_{*}(\Pi)) \;\left(\subset \varinjlim_{J\subset\Pi \text{ open}} H^1(J, \Pi_{\mu}(M^{\theta}_{*}(\Pi)))\right)$$

$\eta + \log(\mathcal{O}^{\times}_{\ddot{y}})$ stable under $\iota$

$\rightsquigarrow \mu_{2\ell}\,\Theta^{\iota\ell}$ stable under $\iota$

the inverse image of $\mathcal{O}^{\mu}(-)$

$= {}_{\infty}\theta(\Pi)^{\iota}$, assoc. $(M^{\theta}_{*}(\Pi))^{\iota}$

In particular, we obtain a fct. alg'm

$\{\mathcal{O}^{\times\mu}(\Pi) \times \{{}_{\infty}\theta(\Pi)^{\iota}/\mathcal{O}^{\mu}(\Pi)\}$

$\mathcal{O}^{\times\mu}(M^{\theta}_{*}(\Pi)) \times \{{}_{\infty}\theta_{env}(M^{\theta}_{*}(\Pi))^{\iota}/\mathcal{O}^{\mu}(M^{\theta}_{*}(\Pi))\}$

splitting

important splitting

$\mathcal{O}^{\times}(M^{\theta}_{*}(\Pi)) \subset \mathcal{O}^{\times}(M^{\theta}_{*}(\Pi)) \cdot {}_{\infty}\theta_{env}(M^{\theta}_{*}(\Pi)) \subset \varinjlim_{J\subset\Pi \text{ open}} H^1(\Pi_{\ddot{y}}(M^{\theta}_{*}(\Pi)))$

$\| \quad \mathcal{O}^{\times}_{\infty}\theta_{env}(M^{\theta}_{*}(\Pi))$

(iii) $(\Pi \curvearrowright \Pi_\mu(\mathcal{M}^\Theta_*(\Pi))\otimes\mathbb{Q}/\mathbb{Z},\ G \curvearrowright \mathcal{O}^{\times\mu}(G),\ \alpha_{\mu\times\mu})$

$\mathcal{R} \to \mathcal{R}$, $e$

[ $\mathcal{M}^\Theta_*(\Pi)$, subsets $(\dagger_{\times\Theta})(\Pi)$, splittings mod $\mathcal{O}^\mu$ $(\dagger_{\mu\Theta})(\Pi)$,

diag. $\Pi_\mu(\mathcal{M}^\Theta_*(\Pi))\otimes\mathbb{Q}/\mathbb{Z} \xrightarrow{\sim} \mathcal{O}^\mu(\mathcal{M}^\Theta_*(\Pi)) \xrightarrow{\sim} \mathcal{O}^\mu(\Pi) \to \mathcal{O}^{\times\mu}(\Pi) \xrightarrow[\text{poly}]{\sim} \mathcal{O}^{\times\mu}(G)$ )

induced by — cycl. rig. — Inn(G)-orbit of isom induced by $\Pi/\Delta \xrightarrow{\sim} G$ full poly

$\Pi_\mu(\mathcal{M}^\Theta_*(\Pi))\otimes\mathbb{Q}/\mathbb{Z} \subset \varinjlim_J H^1(\Pi_{\underline{Y}}(\mathcal{M}^\Theta_*)|_J, \Pi_\mu(\mathcal{M}^\Theta_*))$

is multiradially defined.

$\Delta$

${}_\infty\underline{\Theta}(\Pi)$

mono-an.

$G \curvearrowright \mathcal{O}^{\times\mu}(G)$, $\hat{\mathbb{Z}}^\times$

${}_\infty\underline{\Theta}(\Pi)$

$\Pi_\mu(\mathcal{M}^\Theta_*(\Pi)) \cong (l\Delta_\Theta)(\mathcal{M}^\Theta_*(\Pi))$

cf. [IUTchI, Rem 1.6.2]

$\mu, \Delta$ ~~> vanish in the row

theta fn $\xrightarrow{\text{HA end}}$ theta value — [IUTchIII]

multirad ~~> from a naive pt of view, highly multirad

unirad. <~~ const. monoids & class. cycl. rig. $\hat{\mathbb{Z}}^\times$

i) $\exists$ fct. gp thic algm

$\Pi \mapsto \{(\iota, D)\}(\Pi)$

pointed inversion autom.

$\Delta_{\underline{Y}}(\Pi) := \Delta \cap \Pi_{\underline{Y}}(\Pi)$ - outer autom. of $\Pi_{\underline{Y}}(\Pi)$

"$\pm\sqrt{-1}$"

$\leadsto D \subset \Pi_{\underline{Y}}(\Pi)$: $\Delta_{\underline{Y}}(\Pi)$-conj. class of decomp. subgps

ii) $(\iota, D)$: pt'd inversion autom.

restr. to $D\ (\subset \Pi_{\underline{Y}}(\Pi))$

~~> comm. diag.

$\iota$-inv. part

$\{\mathcal{O}^\times \cdot {}_\infty\underline{\theta}(\Pi)\}^\iota \xrightarrow{\text{restr. to } D} \mathcal{O}^\times(\Pi) \quad (\subset \varinjlim_{J\subset\Pi} H^1(J, (l\Delta_\Theta)(\Pi)))$ cycl. rig

$\{\mathcal{O}^\times \cdot {}_\infty\underline{\theta}_{env}(\mathcal{M}^\Theta_*(\Pi))\}^\iota \xrightarrow{\text{restr. to } D} \mathcal{O}^\times(\mathcal{M}^\Theta_*(\Pi)) \quad (\subset \varinjlim_{J\subset\Pi} H^1(J, \Pi_\mu(\mathcal{M}^\Theta_*(\Pi))))$

$\eta + \log(\mathcal{O}^\times)$ stable under $\iota$

~~> $\mu_{2l}\,\Theta^\iota$ stable under $\iota$

the mono-an image of $\mathcal{O}^\mu(-)$ = ${}_\infty\underline{\theta}(\Pi)^\iota$, ${}_\infty\underline{\theta}_{env}(\mathcal{M}^\Theta_*)$ of std type

In particular, we obtain a fct algm.

$(\dagger_{\mu\Theta})(\Pi)$ ~~> splitting $\{\mathcal{O}^{\times\mu}(\Pi) \times \{{}_\infty\underline{\theta}(\Pi)\}^\iota / \mathcal{O}^\mu(\Pi)\}$, $\{\mathcal{O}^{\times\mu}(\mathcal{M}^\Theta_*(\Pi)) \times \{{}_\infty\underline{\theta}_{env}(\mathcal{M}^\Theta_*(\Pi))\}^\iota / \mathcal{O}^\mu(\mathcal{M}^\Theta_*(\Pi))\}$

const. — "purely radial"

important splitting

## §10.2 Bad Places

$\underline{v} \in \underline{\mathbb{V}}^{\mathrm{bad}}$

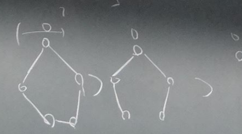
$$\begin{array}{ccccc}
\Pi^{\mathrm{tp}}_{\ddot{\underline{Y}}_{\underline{v}}} & \overset{\mu_2}{\hookrightarrow} & \Pi^{\mathrm{tp}}_{\underline{Y}_{\underline{v}}} & \overset{l\mathbb{Z}}{\hookrightarrow} & \Pi^{\mathrm{tp}}_{\underline{\underline{X}}_{\underline{v}}} =: \Pi_{\underline{v}} \\
\big\downarrow \Delta_\Theta \otimes \mathbb{Z}/l & & \big\downarrow \Delta_\Theta \otimes \mathbb{Z}/l & & \big\downarrow \Delta_\Theta \otimes \mathbb{Z}/l \\
\Pi^{\mathrm{tp}}_{\ddot{Y}_{\underline{v}}} & \overset{\mu_2}{\hookrightarrow} & \Pi_{Y_{\underline{v}}} & \underset{l\mathbb{Z}}{\hookrightarrow} & \Pi^{\mathrm{tp}}_{\underline{X}_{\underline{v}}}
\end{array}$$

[IUTch II, Prop 2.1, Rem 2.1.1, Prop 2.2]

indus

$\iota_{\underline{\underline{X}}} \sim \iota_{\underline{X}}$

$\iota_{\ddot{\underline{Y}}} \sim \iota_{\ddot{Y}}$



$\Gamma_{\underline{X}} \supset \Gamma^{\bullet}_{\underline{X}} \supset \Gamma^{\bullet}_{\underline{X}}$

unique conn. subgp tree, stable under $\iota_{\underline{X}}$, containing vertices of $\Gamma_{\underline{X}}$

unique conn. subgraph, stable under $\iota_{\underline{X}}$ only one vertex, no edge

dual graph of special fiber

$$\begin{array}{ccccc}
\Gamma_{\ddot{\underline{Y}}} & \to & \Gamma_{\underline{Y}} & \to & \Gamma_{\underline{\underline{X}}} \\
\big\downarrow & & \big\| & & \big\downarrow \\
\Gamma_{\ddot{Y}} & \to & \Gamma_{Y} & \to & \Gamma_{\underline{Y}}
\end{array}$$

(iii) $\Big( \Pi \curvearrowright \Pi_\mu(\mathcal{M}^\Theta_*(\Pi)) \otimes \mathbb{Q}/\mathbb{Z},\ G \curvearrowright \mathcal{O}^{\times\mu}(G),\ \alpha_{\mu,\times\mu} \Big)$

$\Big\downarrow$

$\Big( \mathcal{M}^\Theta_*(\Pi)$, subsets $(\dagger_{\times\Theta})(\Pi)$, splittings mod $\mathcal{O}^\mu$ $(\dagger_{\mu\Theta})(\Pi)$,

diag. $\Pi_\mu(\mathcal{M}^\Theta_*(\Pi)) \otimes \mathbb{Q}/\mathbb{Z} \xrightarrow{\sim} \mathcal{O}^\mu(\mathcal{M}^\Theta_*(\Pi)) \xrightarrow{\sim} \mathcal{O}^\mu(\Pi) \xrightarrow{\ } \mathcal{O}^{\times\mu}(\Pi) \xrightarrow[\text{poly}]{\sim} \mathcal{O}^{\times\mu}(G) \Big)$

induced by $\ \ H^1(\Pi_{\underline{X}}(\mathcal{M}^\Theta_*)) \to \Pi_\mu(\mathcal{M}^\Theta_*))$

cycl. rig.

$\mathrm{Im}(\hat{\mathbb{Z}}^\times)$-orbit of isom induced by $\Pi/\Delta \xrightarrow[\text{full poly}]{\sim} G$

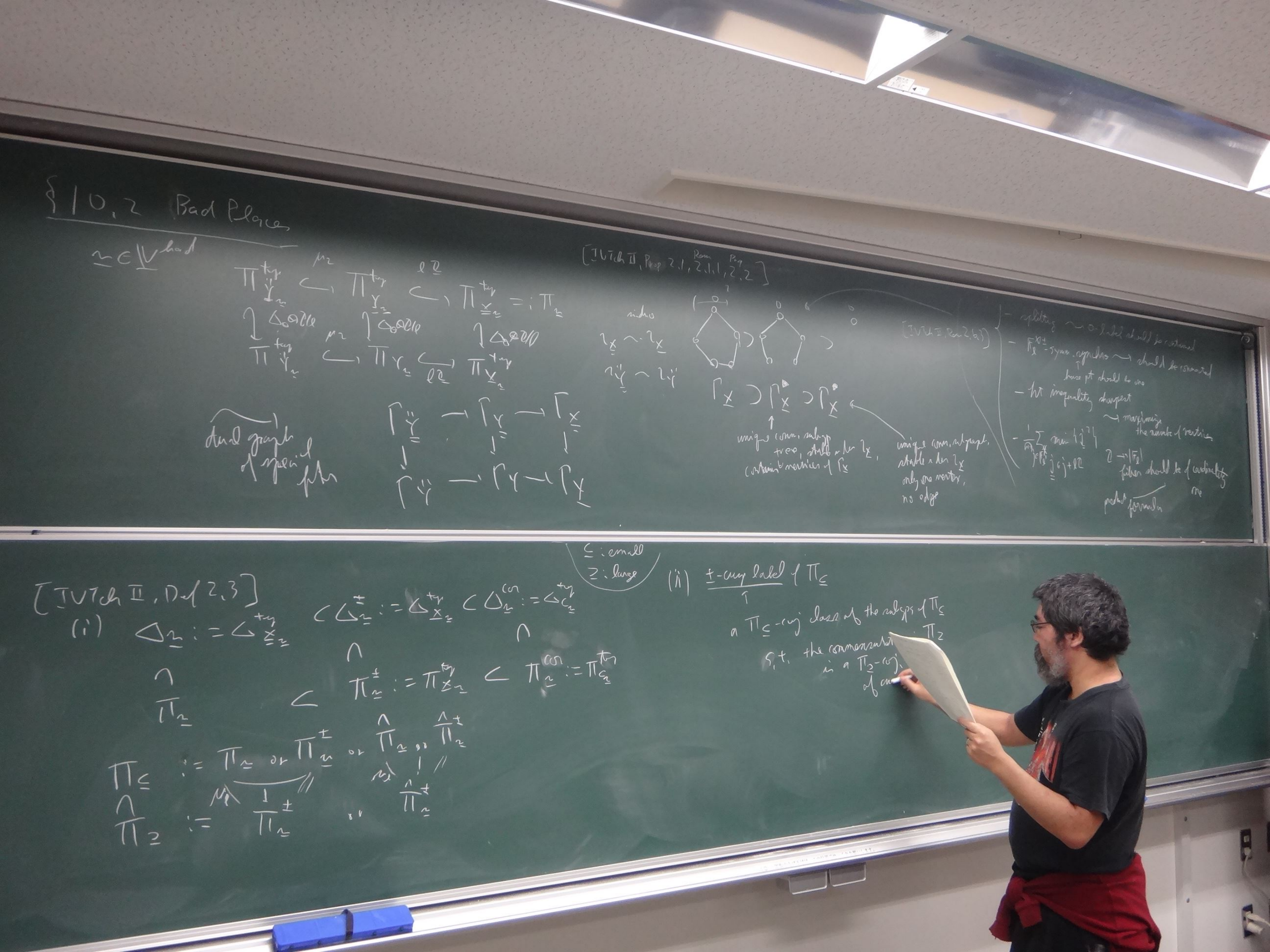
$\mathbb{R}$

$\Delta$

mono-an.

$G \curvearrowright \mathcal{O}^{\times\mu}(G)$

$\hat{\mathbb{Z}}^\times$


§10.2 Bad Places
$\underline{v} \in \underline{\mathbb{V}}^{bad}$
[IUTch II, Prop 2.1, Rem 2.1.1, Prop 2.2]
dual graph of special fiber
unique conn. subgraph, tree, stable under $\iota_{\underline{X}}$, containing vertices of $\Gamma_{\underline{X}}$
unique conn. subgraph, stable under $\iota_{\underline{X}}$, only one vertex, no edge
[IUTch II, Rem 2.6.3]
base pt should be one
ht inequality sharpest
maximize the number of vertices
filter should be of cardinality one
product formula
[IUTch II, Def 2.3]
(ii) ±-conj label of $\Pi_{\subseteq}$
a $\Pi_{\subseteq}$-conj class of the subgps of $\Pi_{\supseteq}$
s.t. the commensurator

[IUTch II, Def 2.3]

(i)

$\Delta_{\underline{\simeq}} := \Delta^{\mathrm{tp}}_{\underline{\underline{X}}_{\underline{\simeq}}} \subset \Delta^{\pm}_{\underline{\simeq}} := \Delta^{\mathrm{tp}}_{\underline{X}_{\underline{\simeq}}} \subset \Delta^{\mathrm{cor}}_{\underline{\simeq}} := \Delta^{\mathrm{tp}}_{C_{\underline{\simeq}}}$

$\cap \qquad \cap \qquad \cap$

$\Pi_{\underline{\simeq}} \subset \Pi^{\pm}_{\underline{\simeq}} := \Pi^{\mathrm{tp}}_{\underline{X}_{\underline{\simeq}}} \subset \Pi^{\mathrm{cor}}_{\underline{\simeq}} := \Pi^{\mathrm{tp}}_{C_{\underline{\simeq}}}$

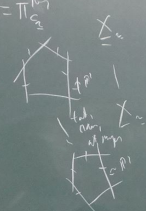
$\le$ : small

$\ge$ : large

$\Pi_{\le} := \Pi_{\underline{\simeq}}$ or $\Pi^{\pm}_{\underline{\simeq}}$ or $\hat{\Pi}_{\underline{\simeq}}$ or $\hat{\Pi}^{\pm}_{\underline{\simeq}}$

$\cap$

$\Pi_{\ge} := \Pi^{\pm}_{\underline{\simeq}}$ or $\hat{\Pi}^{\pm}_{\underline{\simeq}}$




(iii) $\underline{\pm\text{-cusp label}}$ of $\Pi_{\le}$

a $\Pi_{\le}$-conj. class of the cuspidal decomp. gp of $\Pi_{\le}$

s.t. the commensurator in $\Pi_{\ge}$ is a $\Pi_{\ge}$-conj. class of cuspidal decomp. gp

$\mathrm{LabCusp}^{\pm}(\Pi_{\le})$

$H \subset G$ closed subgp

$C_G(H) := \{ g \in G \mid gHg^{-1} \cap H \subset H, gHg^{-1}$ of fin. index $\}$

$\Pi_{\le} = \Pi_{\underline{\simeq}} \Rightarrow \mathrm{LabCusp}^{\pm}(\Pi_{\le}) \cong \mathrm{LabCusp}(\Pi_{D})$

$\mathbb{F}_l^{\times} \curvearrowright \ {}^{\mathrm{t}}\eta^{0}_{\underline{\simeq}}$ - zero elt

${}^{\mathrm{t}}\eta^{\pm}_{\underline{\simeq}}$ $\pm$-can. gen. well def up to $\pm 1$

(iv) $t \in \mathrm{LabCusp}^{\pm}(\Pi_{\underline{\simeq}})$

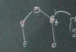
$\rightsquigarrow$ unique vertex of $\Gamma^{\bullet}_{\underline{X}}$

$\cup$

$\Gamma^{\bullet t}_{\underline{X}}$ no edges

$\rightsquigarrow \Pi_{\underline{\simeq}\bullet t} \le \Pi_{\underline{\simeq}\bullet} \le \Pi_{\underline{\simeq}}$

decomp. gp well def up to $\Pi_{\underline{\simeq}\bullet}$-conj.




$\Delta^{\mathrm{tp}}_{X} \times \Delta^{\mathrm{tp}}_{\mathcal{H}}$ - prof. part $\Sigma = \{ l \}$

$\Delta^{\mathrm{tp}}_{\mathcal{G}}$ dual graph

special fiber of the stable model of $X$

decomp sp of $\mathcal{H}$ $\to$ $\Delta^{\text{tmp}}_{X} \times \Delta^{\text{tmp}}_{\mathcal{H}}$ $\leftarrow$ mod. part

$\Delta^{\text{tmp}}_{\mathcal{G}}$ sub-graph $\Sigma = \{\ell\}$

mod. part

Brauer

special fiber of the stable model of $X$

$H \underset{\text{open}}{\subset} G$, $I \underset{\text{open}}{\subset} G$

$I \subset H \subset G$ factorization

tmp cusp vs prof cusp.

$$\Pi^{\pm}_{\underline{\nu}\bullet\dagger} := N_{\Pi^{\pm}_{\simeq}}(\Pi_{\underline{\nu}\bullet\dagger})$$

(u) $\Pi_{\subseteq} := \Pi^{\pm}_{\simeq}$ or $\hat{\Pi}^{\pm}_{\simeq}$ $\quad X_{\simeq}$

$\cap$

$\Pi_{\supseteq} := \Pi^{\text{cor}}_{\simeq}$ or $\hat{\Pi}^{\text{cor}}_{\simeq}$ $\quad C_{\simeq}$ $\quad \mathbb{F}^{\rtimes\pm}_{\ell}$

$\Pi_{\simeq} \subset \Pi_{\subseteq}$

$\rightsquigarrow$ images in $\mathrm{LabCusp}^{\pm}(\Pi_{\subseteq})$ $\quad \mathbb{F}^{\pm}_{\ell}$-torsor

of $\mathrm{LabCusp}^{\pm}(\Pi_{\simeq})$ $\quad \Pi_{\supseteq}/\Pi_{\subseteq} \curvearrowright \Pi_{\subseteq}$

$\mathbb{F}^{\pm}_{\ell}$-torsor $\quad$ preserves

$\Pi_{\supseteq}/\Pi_{\subseteq} \xrightarrow{\sim} \mathbb{F}^{\rtimes\pm}_{\ell}$

outer isom.

[IUTchII (?)]

$\square = \dagger$ or $\ddagger$

$\Delta^{(\text{II})}_{\simeq\square} := \Delta_{\simeq} \cap \Pi^{(\text{II})}_{\simeq\square}$

$\Pi_{\simeq\square} := \Pi_{\simeq\square} \cap \Pi^{\text{tmp}}_{I_{\simeq}}$

$(\Delta) \quad (\Delta)$

(i) $t \in \mathrm{LabCusp}^{\pm}(\Pi_{\simeq})$

$I_{t} \subseteq \Pi_{\simeq}$ cuspidal inertia

$(\Delta_{\simeq\square})$

Consider the following
sets of conj. classes;

$\{ I_A^{\gamma_1} \}_{\gamma_1 \in \hat{\Pi}^{\pm}_{\simeq}} = \{ I_*^{\gamma_1} \}_{\gamma_1 \in \hat{\Delta}^{\pm}_{\simeq}}$,

$\{ \Pi_{\simeq\square}^{\gamma_2} \}_{\gamma_2 \in \hat{\Pi}^{\pm}_{\simeq}} = \{ \Pi_{\simeq\square}^{\gamma_2} \}_{\gamma_2 \in \hat{\Delta}^{\pm}_{\simeq}}$,

$\{ \Pi_{\simeq\square}^{\pm\gamma_3} \}_{\gamma_3 \in \hat{\Pi}^{\pm}_{\simeq}} = \{ \Pi_{\simeq\square}^{\pm\gamma_3} \}_{\gamma_3 \in \hat{\Delta}^{\pm}_{\simeq}}$

$\boxed{+}$

the following is equiv:

(a). $\gamma' \in \Delta^{\pm}_{\simeq\square}$

(b). $I_A^{\gamma\gamma'} \subset \Pi_{\simeq\square}^{\gamma}$

(c). $I_*^{\gamma\gamma'} \subset (\Pi^{\pm}_{\simeq\square})^{\gamma}$

(ii) $\delta := \gamma\gamma' \in \hat{\Delta}^{\pm}_{\simeq}$

Assume $I_*^{\delta} = I_*^{\gamma\gamma'} \subseteq \Pi_{\simeq\square}^{\gamma} = \Pi_{\simeq\square}^{\gamma}$

$\rightsquigarrow$ A def's

(a). decomp. gp $D_*^{\delta} := N_{\Pi^{\delta}_{\simeq}}(I_*^{\delta}) \subseteq \Pi^{\delta}_{\simeq\ddot{\square}}$

(b). decomp. gp of $\mu_-$-triangle. $D^{\delta}_{\mu_-} \subset \Pi^{\delta}_{\simeq\ddot{\square}}$

well-def up to $(\Pi^{\pm}_{\simeq\square})^{\delta}$-conj.

(c). decomp. gp $D^{\delta}_{*|\mu_-} \subset \Pi^{\delta}_{\simeq\ddot{\square}}$

well-def up to $(\Pi^{\pm}_{\simeq\square})^{\delta}$-conj.

$\mu_2$

(iii) $\underline{\mathbb{F}_\ell^{\times\pm}\text{-symmetry}}$ — one base pt $\to \mathbb{F}_\ell^{\times\pm}$-synchro $\to$ "good diag." $\to$ base, core on diag $\{0, \langle\ \rangle\} = \Delta$

$\square = *$

condition of (ii)(a),(c)

is compat. w/.

conj. by $\delta \in \hat{\Pi}^{cor}_{\simeq}$

$(\hat{\Pi}^{cor}_{\simeq} / \hat{\Pi}^{\pm}_{\simeq} \xrightarrow{\sim} \mathbb{F}_\ell^{\times\pm})$

$H \subset G$, $I \subset G$
out out

$I \subset H \subset G$ factorization
out out

top. cusp vs prof. cusp.

$\Delta^{top} \times \Delta^{tem}_{\mathcal{X}}$ — prof. part $\Sigma = \{\ell\}$

$\Delta_{\mathcal{X}}^{top}$ $\Delta_{\mathcal{G}}$ sub-graph

special fiber of the stable model of $X$

$\Pi^{\pm}_{\simeq\square *} := N_{\Pi^{\pm}_{\simeq}}(\Pi_{\simeq\square *})$

(v) $\Pi_{\subseteq} := \Pi^{\pm}_{\simeq}$ or $\hat{\Pi}^{\pm}_{\simeq}$ $\quad X_{\simeq}$ $\Big) \mathbb{F}_\ell^{\times\pm}$

$\Pi_{\supseteq} := \Pi^{cor}_{\simeq}$ or $\hat{\Pi}^{cor}_{\simeq}$ $\quad C_{\simeq}$

$\Pi_{\simeq} \subset \Pi_{\subseteq}$

$\rightsquigarrow$ images in $\mathrm{LabCusp}^{\pm}(\Pi_{\subseteq})$ $\curvearrowleft$ $\mathbb{F}_\ell^{\pm}$-torsor

of $\mathrm{LabCusp}^{\pm}(\Pi_{\simeq})$

$\mathbb{F}_\ell^{\pm}$-torsor

$\Pi_{\supseteq}/\Pi_{\subseteq} \curvearrowright \Pi_{\subseteq}$

preserves

$\Pi_{\supseteq}/\Pi_{\subseteq} \xrightarrow{\sim} \mathbb{F}_\ell^{\times\pm}$ outer isom.

[IUTchII, Cor 2.4]

$\square = *$ or $\blacktriangle$

$\Delta^{(\pm)}_{\simeq\square} := \Delta_{\simeq} \cap \Pi^{(\pm)}_{\simeq\square}$

$\Pi_{\simeq\ddot{\square}} := \Pi_{\simeq\square} \cap \Pi^{\mu_2}_{\ddot{Y}_{\simeq}}$

$(\Delta)$ $\quad(\Delta)$

(i) $* \in \mathrm{LabCusp}^{\pm}(\Pi_{\simeq})$

$I_* \subseteq \Pi_{\simeq}$ cuspidal inertia
$(\Delta_{\simeq\square})$

Consider the following sets of conj. classes;

$$\{ I_A^{\gamma_1} \}_{\gamma_1 \in \hat{\Pi}^{\pm}_{\underline{\ }}} = \{ I_*^{\gamma_1} \}_{\gamma_1 \in \hat{\Delta}^{\pm}_{\underline{\ }}},$$

$$\{ \Pi_{\underline{\ }\Box}^{\gamma_2} \}_{\gamma_2 \in \hat{\Pi}^{\pm}_{\underline{\ }}} = \{ \Pi_{\underline{\ }\Box}^{\gamma_2} \}_{\gamma_2 \in \hat{\Delta}^{\pm}_{\underline{\ }}},$$

$$\{ \Pi^{\pm\gamma_3}_{\sim\Box} \}_{\gamma_3 \in \hat{\Pi}^{\pm}_{\underline{\ }}} = \{ \Pi^{\pm\gamma_3}_{\underline{\ }\Box} \}_{\gamma_3 \in \hat{\Delta}^{\pm}_{\underline{\ }}}$$

$\boxed{+}$

the following is equiv;

(a). $\gamma' \in \Delta^{\pm}_{\underline{\ }\Box}$

(b). $I_A^{\gamma\gamma'} \subset \Pi^{\gamma}_{\underline{\ }\Box}$

(c). $I_*^{\gamma\gamma'} \subset (\Pi^{\pm}_{\underline{\ }\Box})^{\gamma}$

(ii) $\delta := \gamma\gamma' \in \hat{\Delta}^{\pm}_{\underline{\ }}$

Assume $I_*^{\delta} = I_*^{\gamma\gamma'} \subseteq \Pi^{\gamma}_{\underline{\ }\Box} = \Pi^{\delta}_{\underline{\ }\Box}$

$\rightsquigarrow$ a def'n

(a). decomp. gp $D_*^{\delta} := N_{\Pi^{\delta}_{\underline{\ }}}(I_*^{\delta}) \subseteq \Pi^{\delta}_{\underline{\ }\Box}$

(b). decomp. gp of $\mu_-$-trans. $D_{\mu_-}^{\delta} \subset \Pi^{\delta}_{\underline{\ }\Box}$ well-def'd up to $(\Pi^{\pm}_{\underline{\ }\Box})^{\delta}$-conj.

(c). decomp. gp $D_{*,\mu_-}^{\delta} \subset \Pi^{\delta}_{\underline{\ }\Box}$ well-def'd up to $(\Pi^{\pm}_{\underline{\ }\Box})^{\delta}$-conj.

$\mu_2$

(iii) $\mathbb{F}_l^{\times\pm}$-symmetry

one base pt

$\to \mathbb{F}_l^{\times\pm}$-synchro

"good thing"

function

$\Box = 0, \dagger$

condition of (ii)(a), (c)

is compat. w/.

conj. by $\delta \in \hat{\Pi}^{cm}_{\underline{\ }}$

$(\hat{\Pi}^{cm}_{\underline{\ }} / \hat{\Pi}^{\pm}_{\underline{\ }} \xrightarrow{\sim} \mathbb{F}_l^{\times\pm})$

$= \Delta$

taking conj. VS poly. cy

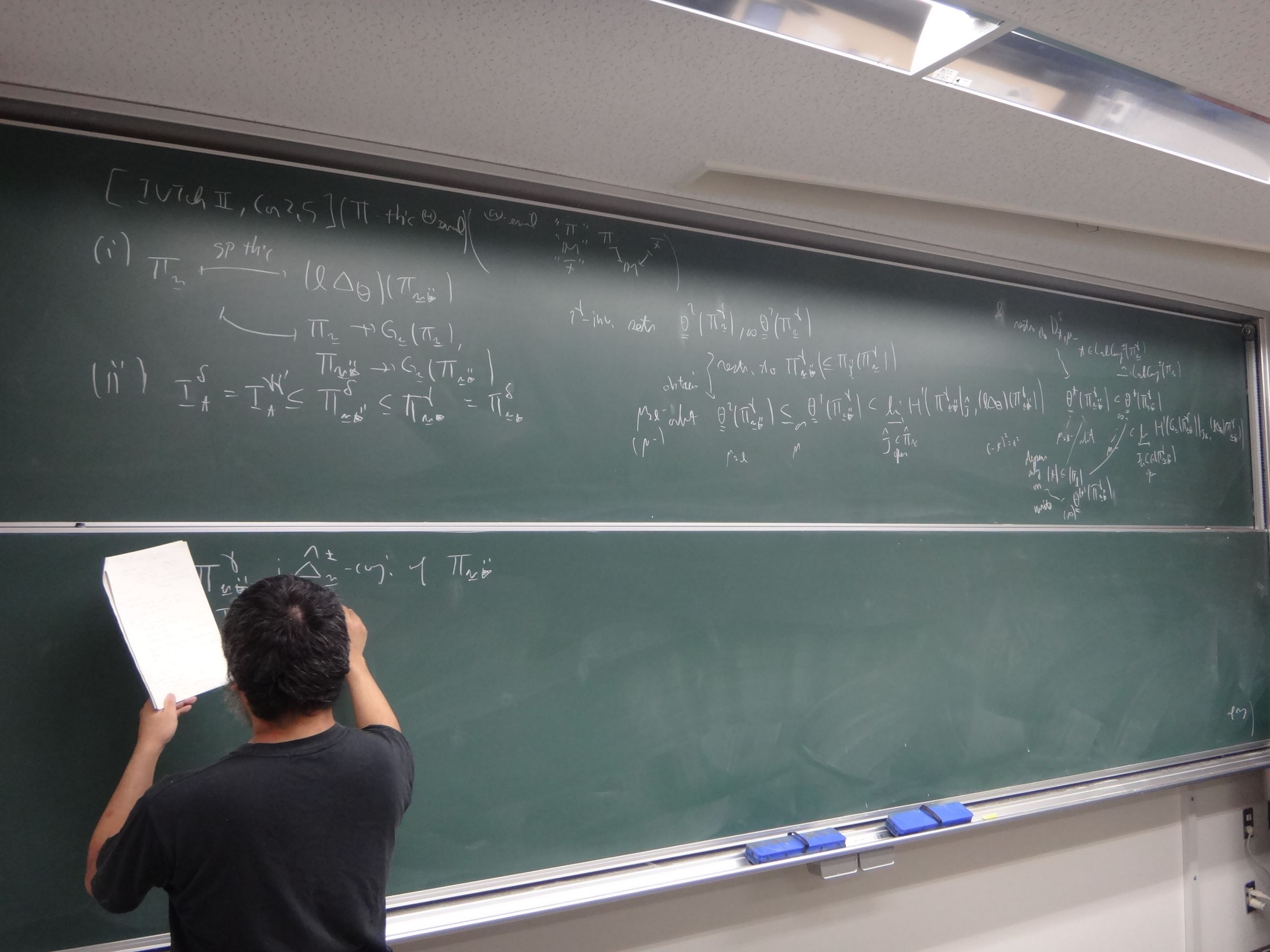
Thb.6

[ IUTch II, Cor 2.5 ]

("$\Pi$" $\Pi$ $\overline{X}$

"$M$" $I_M$

"$\overline{X}$")


[IUTch II, Cor 2.5]
sp th'c
obtain
open

(iii) $\Pi^{\gamma}_{\underline{v}\heartsuit}$ : $\hat{\underline{\Delta}}^{\pm}$-conj. of $\Pi_{\underline{v}\heartsuit}$

$I^{\delta}_{t}$ : $\hat{\underline{\Delta}}^{\pm}$-conj. of $I_{t} \subset \Pi^{\gamma}_{\underline{v}\heartsuit}$ s.t.

$\rightsquigarrow \underline{\underline{\theta}}^{|t|}(\Pi^{\gamma}_{\underline{v}\heartsuit})$

$\rightsquigarrow$ collection of $\mu_{2\ell}$-($\mu$-)orbits

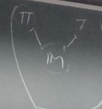
$\left\{ \underset{(\infty)}{\underline{\underline{\theta}}}^{|t|}(\Pi^{\gamma}_{\underline{v}\heartsuit}) \right\}_{|t| \in |\mathbb{F}_{\ell}|}$

functorial w.r.t. $\Pi_{\underline{v}}$

compat. w/ indep. conj. actions of $\hat{\underline{\Delta}}^{\pm}_{\underline{v}}$ $\cap$ $\{I^{\gamma_1}_{t}\}_{\gamma_1 \in \hat{\Pi}^{\pm}_{\underline{v}}}$ & $\{\Pi^{\gamma_2}_{\underline{v}\heartsuit}\}_{\gamma_2 \in \hat{\Pi}^{\pm}_{\underline{v}}}$

set $\cap$

$\{I^{\gamma_1}_{t}\}_{\gamma_1 \in \hat{\underline{\Delta}}^{\pm}_{\underline{v}}}$ $\quad$ $\{\Pi^{\gamma_2}_{\underline{v}\heartsuit}\}_{\gamma_2 \in \hat{\underline{\Delta}}^{\pm}_{\underline{v}}}$




[IUTch II, Cor 2.5]

(i') $\underbrace{\Pi_{\underline{v}\heartsuit}}_{\text{F-conj. of}} \mapsto G_{\underline{v}}(\Pi_{\underline{v}\heartsuit})$

obtain $\theta^{\times}(\Pi^{\gamma}_{\underline{v}\heartsuit}) \subseteq \theta^{\times}\cdot\underline{\underline{\theta}}^{\gamma}(\Pi^{\gamma}_{\underline{v}\heartsuit}) \subset \theta^{\times}_{\infty}\underline{\underline{\theta}}^{\gamma}(\Pi^{\gamma}_{\underline{v}\heartsuit})$

$\subset \varinjlim_{\hat{J} \subset \hat{\Pi}_{\underline{v}}\text{ op}} H^1(\Pi^{\gamma}_{\underline{v}\heartsuit}|_{\hat{J}}, \ell\Delta_{\Theta}(\Pi^{\gamma}_{\underline{v}\heartsuit}))$

(ii) $t = 0$: $\underset{(\infty)}{\underline{\underline{\theta}}}^{t}(\Pi^{\gamma}_{\underline{v}\heartsuit})$ $= 1$

in $\mu_{2\ell}$-($\mu$-)orbit of id elt.

$D^{\delta}_{t,\mu\text{-}}$ $t=0$ $\rightsquigarrow$ splitting of $\theta^{t,\mu}(\Pi^{\gamma}_{\underline{v}\heartsuit}) \to \{{}_{\infty}\underline{\underline{\theta}}^{\gamma}(\Pi^{\gamma}_{\underline{v}\heartsuit})/\theta^{\mu}(\Pi^{\gamma}_{\underline{v}\heartsuit})\}$

of $\theta^{\times}_{\infty}\underline{\underline{\theta}}(\Pi^{\gamma}_{\underline{v}\heartsuit})/\theta^{\mu}(\Pi^{\gamma}_{\underline{v}\heartsuit})$

splittings by theta fct.

compat. w/ splittings in Cor 1.12 (iii)

[IUTch II, Cor 2.8] (M-th'c $\Theta$-eval)

$\Pi_{\underline{X}}^{\gamma} \rightsquigarrow$ mono-theta env. $(\mathcal{M}_{*}^{\Theta})^{\gamma}$

(i') cycl. rig. $(\ell\Delta_{\Theta})((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma}) \cong \Pi_{\mu}((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma})$

$(\ell\Delta_{\Theta})((\mathcal{M}_{*}^{\Theta})^{\gamma}) \cong \Pi_{\mu}((\mathcal{M}_{*}^{\Theta})^{\gamma})$

replace $(\ell\Delta_{\Theta})(-)$ by $\Pi_{\mu}(-)$

$\rightsquigarrow$ $\gamma$-inv. subset $\underset{(\omega)}{\underline{\Theta}}^{\gamma}(\Pi_{\underline{X}}^{\gamma}) \subset \underset{(\omega)}{\underline{\Theta}}(\Pi_{\underline{X}}^{\gamma})$

$\rightsquigarrow$ $\underset{(\omega)}{\underline{\Theta}}^{\gamma}_{\mathrm{env}}((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma}) \subset \underset{(\omega)}{\underline{\Theta}}_{\mathrm{env}}((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma})$

def'l

$\Pi_{\mathcal{M}_{*}^{\Theta}} := \varprojlim_{N}(\Pi_{\mathcal{M}_{N}^{\Theta}}) \rightarrow \Pi_{\underline{X}}(\mathcal{M}_{*}^{\Theta}) \cong \Pi_{\underline{X}}$

$\cup \qquad \Delta_{\mathcal{M}} = \Pi_{\mu}(\mathcal{M}_{*}^{\Theta}) \qquad \cup \qquad \cup$

$I_{\mathcal{M}} \cong \Pi_{\underline{X}}^{\mathrm{tmp}} \cup$

$\Pi_{\mathcal{M}_{*\flat}^{\Theta}}$ $\llcorner$ dd $\qquad \Pi_{\underline{X}_{\flat}}$

$\cup \qquad \llcorner$ dd $\qquad \cup$

$\Pi_{\mathcal{M}_{*\ddot{\flat}}^{\Theta}} \qquad \Pi_{\underline{X}_{\ddot{\flat}}}$

$\Pi_{\mu}(\mathcal{M}_{*\ddot{\flat}}^{\Theta})$, $(\ell\Delta_{\Theta})(\mathcal{M}_{*\ddot{\flat}}^{\Theta})$, $\Pi_{\ddot{\flat}}(\mathcal{M}_{*\ddot{\flat}}^{\Theta})$,

$G_{\underline{k}}(\mathcal{M}_{*\ddot{\flat}}^{\Theta})$ corresp. subquot of $\Pi_{\mathcal{M}_{*\ddot{\flat}}^{\Theta}}$

$\rightsquigarrow$

restr. to $\Pi_{\underline{X}_{\ddot{\flat}}}((\mathcal{M}_{*}^{\Theta})^{\gamma})$ $(\omega)$ $\underline{\Theta}^{\gamma}_{\mathrm{env}}((\mathcal{M}_{*}^{\Theta})^{\gamma}) \subset \varinjlim_{\hat{J} \subset \hat{\Pi}_{\underline{X}} \text{ opn}} H^{1}(\Pi_{\ddot{\flat}}((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma})|_{\hat{J}}, \Pi_{\mu}(-))$

$A \in \mathrm{Sub}\mathrm{Grp}^{\pm}(\Pi_{\underline{X}}^{\gamma})$, restr. $A \cup D_{\gamma,\mu}$

$\mu_{2l}$-($\mu$-) orbits

$(\omega)$ $\underline{\Theta}^{\gamma}_{\mathrm{env}}((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma}) \subset \varinjlim_{\hat{J}_G \subset G_{\underline{k}}((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma}) \text{ opn}} H^{1}(G_{\underline{k}}((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma})|_{\hat{J}}, \Pi_{\mu}(-))$

depend on $|t| \in |\mathbb{F}_{l}|$ $(\omega)$ $\underline{\Theta}^{|t|}_{\mathrm{env}}((\mathcal{M}_{*\ddot{\flat}}^{\Theta})^{\gamma})$

notation

(i') $\Pi_{\mu}$

(ii) $\Pi_{\mu\text{-}\ddot{\ }}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma}) : \hat{\Delta}^{t}_{\underline{\ }}$-conj. of $\Pi_{\mu\ddot{\ }}(\mathcal{M}^{\Theta}_{*\ddot{\ }})$

$I^{\delta}_{*}$ : $\hat{\Delta}^{t}_{\underline{\ }}$-conj. of $I_{*} \subset \Pi^{\gamma}_{\underline{\ }\ddot{\ }}(\ldots)$

$\rightsquigarrow$ formal w.r.t. $\mathcal{M}^{\Theta}_{*}$ $\left\{ \underset{(\infty)}{\underline{\theta}}{}^{|t|}_{\mathrm{env}}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{t}) \right\}_{|t| \in |\mathbb{F}_l|}$

conj. w/ $\hat{\Delta}^{t}_{\underline{\ }}$-conj. indep. $\curvearrowright \{ I^{\gamma_1}_{*} \}_{\gamma_1 \in \hat{\Delta}^{t}_{\underline{\ }}}$

(iii) $* = 0$ by Kum. rig. $(l\Delta_{\Theta})(-1) \overset{\sim}{\to} \Pi_{\mu}(-1)$ $\curvearrowright \{ I_{*\ddot{\ }}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma_2}) \}_{\gamma_2}$

splitting of $\mathcal{O}^{\times} \cdot \underset{(\infty)}{\underline{\theta}}{}^{\gamma}_{\mathrm{env}}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma}) / \mathcal{O}^{\mu}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma})$

$\mathcal{O}^{\times\mu}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma}) \times \left\{ \underset{(\infty)}{\underline{\theta}}{}^{\gamma}_{\mathrm{env}}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma} / \mathcal{O}^{\mu}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma}) \right\}$

comp. w/ the splitting of Cor 1.12 in thm 1.10

[IUTch II, Cor 2.9] (bi-cyc. thick bi-coric)

(cycl. rig. (CFT) $\rightsquigarrow$ unimod.) $\left\{ \begin{array}{l} \mu_{\widehat{\mathbb{Z}}}(G_k(\Pi_{\underline{\ }})) \overset{\sim}{\to} (l\Delta_{\Theta})(\Pi_{\underline{\ }}) \\ \mu^{\wedge}_{\widehat{\mathbb{Z}}}(G_k(\Pi^{\gamma}_{*\ddot{\ }})) \overset{\sim}{\to} (l\Delta_{\Theta})(\Pi^{\gamma}_{\underline{\ }\ddot{\ }}) \end{array} \right.$

$\underset{(\infty)}{\underline{\theta}}{}_{\mathrm{env}} \rightsquigarrow \underset{(\infty)}{\underline{\theta}}{}_{\mathrm{bs}}$

$\underset{(\infty)}{\underline{\theta}}{}^{\wedge}_{\mathrm{env}} \rightsquigarrow \underset{(\infty)}{\underline{\theta}}{}^{\wedge}_{\mathrm{bs}}$

$\underset{(\infty)}{\underline{\theta}}{}^{|t|}_{\mathrm{env}} \rightsquigarrow \underset{(\infty)}{\underline{\theta}}{}^{|t|}_{\mathrm{bs}}$

splitting.

$\Pi$ $\mathcal{M}$ $\mathcal{J}$

Kummer for Frob-like

theta fct. theta monoids

theta values Gaussian monoids

[IUTch II, Cor 2.8] ($\mathcal{M}$-thic $\Theta$-and) $\gamma$

$\Pi^{\gamma}_{\underline{\ }} \rightsquigarrow$ mono-theta env. $(\mathcal{M}^{\Theta}_{*})^{\gamma}$

(i) cycl. rig. $(l\Delta_{\Theta})((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma}) \overset{\sim}{\to} \Pi_{\mu}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma})$

$(l\Delta_{\Theta})((\mathcal{M}^{\Theta}_{*})^{\gamma}) \overset{\sim}{\to} \Pi_{\mu}((\mathcal{M}^{\Theta}_{*})^{\gamma})$

$(l\Delta_{\Theta})((\mathcal{M}^{\Theta}_{*})^{\gamma})$

replace $(l\Delta_{\Theta})(-1)$ by $\Pi_{\mu}(-1)$

$\underset{(\infty)}{\underline{\theta}}{}^{\gamma}(\Pi^{\gamma}_{\underline{\ }}) \subset \underset{(\infty)}{\underline{\theta}}(\Pi^{\gamma}_{\underline{\ }})$

$\rightarrow$ $\gamma$-ind. subset $\underset{(\infty)}{\underline{\theta}}{}^{\gamma}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma}) \subset \underset{(\infty)}{\underline{\theta}}{}_{\mathrm{env}}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma})$

$\underset{(\infty)}{\underline{\theta}}{}^{\gamma}_{\mathrm{env}}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma})$

$\Pi_{\mathcal{M}^{\Theta}_{*}} := \varprojlim_{N} (\Pi_{\mathcal{M}^{\Theta}_{N}}) \twoheadrightarrow \Pi_{\underline{\ }}(\mathcal{M}^{\Theta}_{*}) \overset{\sim}{=} \Pi_{\underline{\ }}$

$\cup$ $\quad$ ker $= \Pi_{\mu}(\mathcal{M}^{\Theta}_{*})$ $\quad$ $I_{*} \overset{\sim}{=} \Pi^{\mathrm{tmp}}_{\underline{\ }\ddot{\ }} \cup \Pi_{\underline{\ }\ddot{\ }}$

$\Pi_{\mathcal{M}^{\Theta}_{*\ddot{\ }}}$ $\llcorner$ def

$\cup$ $\llcorner$ def

$\Pi_{\mathcal{M}^{\Theta}_{*\ddot{\ }}}$, $(l\Delta_{\Theta})(\mathcal{M}^{\Theta}_{*\ddot{\ }})$, $\Pi_{\ddot{\ }}(\mathcal{M}^{\Theta}_{*\ddot{\ }})$,

$\Pi_{\mu}(\mathcal{M}^{\Theta}_{*\ddot{\ }})$, $G_k(\mathcal{M}^{\Theta}_{*\ddot{\ }})$ corresp. subquot. of $\Pi_{\mathcal{M}^{\Theta}_{*\ddot{\ }}}$

$\rightsquigarrow$ $\underset{(\infty)}{\underline{\theta}}{}^{\gamma}_{\mathrm{env}}((\mathcal{M}^{\Theta}_{*})^{\gamma})$

notation, $\Pi_{\underline{\ }\ddot{\ }}((\mathcal{M}^{\Theta}_{*})^{\gamma})$ $\subset \varinjlim_{J \subset \Pi_{\underline{\ }}} H^{1}(\Pi_{\underline{\ }\ddot{\ }}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma})|_{J}, \Pi_{\mu}(\ldots))$

$\Delta\text{-}\mathrm{Aut}(\Pi^{\gamma}_{\underline{\ }})$, notation, $\mathrm{AutD}_{\mathrm{pr}}$

$\mu_{2l}$-(mu-)orbit $\underset{(\infty)}{\underline{\theta}}{}^{\gamma}_{\mathrm{env}}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma}) \subset \varinjlim_{J_0 \subset G_k(\mathcal{M}^{\Theta}_{*\ddot{\ }})} H^{1}(G_k((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma})|_{J_0}, \Pi_{\mu}(\ldots))$

depend on $|t| \in |\mathbb{F}_l|$ $\underset{(\infty)}{\underline{\theta}}{}^{|t|}_{\mathrm{env}}((\mathcal{M}^{\Theta}_{*\ddot{\ }})^{\gamma})$

orbit

(ii) $\Pi_{\mu\ddot{\circledast}}((\mathbb{M}^{\Theta}_{*\ddot{\circledast}})^{\gamma}) : \hat{\Delta}^{\pm}_{\kappa}$-orbit of $\Pi_{\mu\ddot{\circledast}}(\mathbb{M}^{\Theta}_{*\ddot{\circledast}})$

$I^{\delta}_{*}$ : $\hat{\Delta}^{\pm}$-orbit of $I_{*} \subset \Pi^{\gamma}_{\mu\ddot{\circledast}}(\ldots)$

$\rightsquigarrow$ {$\Theta^{|t|}_{\underset{(\infty)}{=}\mathrm{env}}((\mathbb{M}^{\Theta}_{*\ddot{\circledast}})^{\gamma})^{\zeta}$}$_{|t| \in |\mathbb{F}_{l}|}$

fit. w.r.t. $\mathbb{M}^{\Theta}_{*}$

comp. w/ $\hat{\Delta}^{\pm}$-orbit $\supseteq \{ I_{A}^{\gamma_1} \}_{\gamma_1 \in \hat{\Delta}^{\pm}}$

indep.

(iii) $A = 0$ by ... rig $(\ell\Delta_{\Theta})(-)$ ... $\Pi_{\mu}(-)$ $\supseteq \{ I_{\mu\ddot{\circledast}}((\mathbb{M}^{\Theta}_{*\ddot{\circledast}})^{\gamma_2}) \}_{\gamma_2}$

splitting of $\mathcal{O}^{\times}\cdot \Theta^{?}_{\underset{(\infty)}{=}\mathrm{env}}((\mathbb{M}^{\Theta}_{*\ddot{\circledast}})^{\gamma}) / \mathcal{O}^{\mu}((\mathbb{M}^{\Theta}_{*\ddot{\circledast}})^{\gamma})$

$\mathcal{O}^{\times\mu}((\mathbb{M}^{\Theta}_{*\ddot{\circledast}})^{\gamma}) \times \{ \Theta^{?}_{\underset{(\infty)}{=}\mathrm{env}}((\mathbb{M}^{\Theta}_{*\ddot{\circledast}})^{\gamma} / \mathcal{O}^{\mu}((\mathbb{M}^{\Theta}_{*\ddot{\circledast}})^{\gamma}) \}$

comp. w/ the splitting of ... theta fct.

[IUTchII, Cor 2.9] (bs-Gal. thr. Θ-env.)

$(\mathrm{Syl}\ \mathrm{Rig}(GT) \longrightarrow \{ \mathcal{M}^{\circledast}_{\Theta}(G_{v}(\Pi_{v})) \xrightarrow{\sim} (\ell\Delta_{\Theta})(\Pi_{v})$

$\mathcal{M}^{\circledast}_{\Theta}(G_{v}(\Pi^{\gamma}_{\ddot{\circledast}})) \xrightarrow{\sim} (\ell\Delta_{\Theta})(\Pi^{\gamma}_{\ddot{\circledast}})$

univ.

$(\infty)\underline{\Theta}_{\mathrm{env}} \sim (\infty)\underline{\Theta}_{bs}$

$(\infty)\underline{\Theta}^{A}_{\mathrm{env}} \sim (\infty)\underline{\Theta}^{A}_{bs}$

$(\infty)\underline{\Theta}^{|t|}_{\mathrm{env}} \sim (\infty)\underline{\Theta}^{|t|}_{bs}$

splitting

$\Pi$ — $\mathbb{M}$ — $\mathcal{F}$

Kummer for Frob-like

theta fct. — theta monoids

theta values — Gaussian monoids

$\text{theta}_{\Theta}\ (\text{Frob-like}) \xleftarrow{\text{Kum}} (\text{et-like})$

$\text{Gaussian}\ (\text{Frob-like}) \xleftarrow{} (\text{et-like})$

... alg'n.

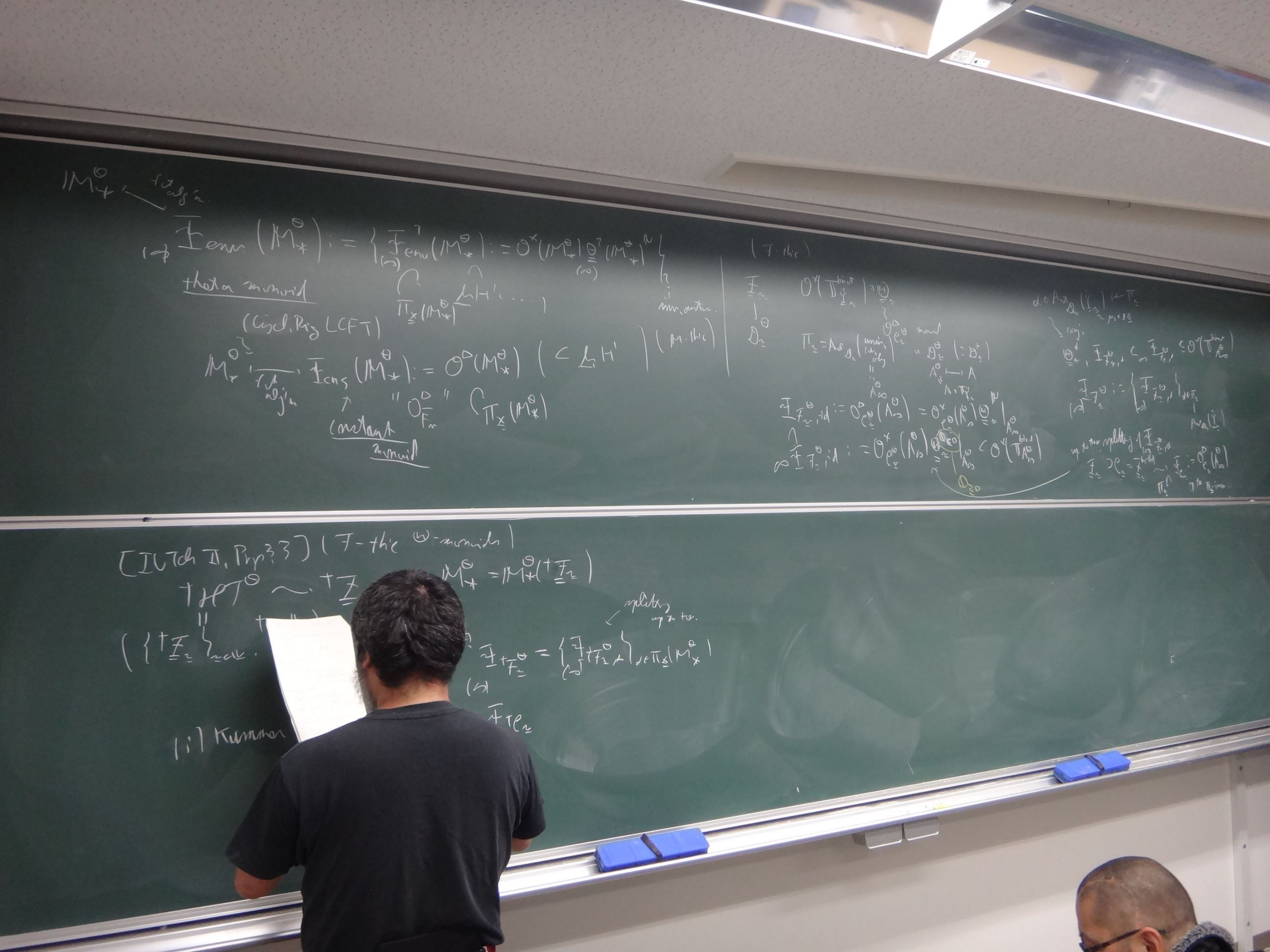
$\overline{\mathcal{I}}\ldots) := \{ \mathcal{F}^{?}_{\underset{(\infty)}{\mathrm{env}}}(\mathbb{M}^{\Theta}_{*}) := \mathcal{O}^{\times}(\mathbb{M}^{\Theta}_{*}) \cdot \underset{(\infty)}{\underline{\Theta}^{?}}(\mathbb{M}^{\Theta}_{*})^{\mu} \}$

mon. anti...

$\cap$

$\mathcal{L}H^{1}(\ldots)$

the...


theta monoid
(Cycl. Rig LCFT)
constant monoid
(M-thic)
(F-thic)
[IUTch II, Prp 3.3] (F-thic Θ-monoids)
splitting up to tor.
(i) Kummer

[IUTch II, Prop. 3.3] ($\mathcal{F}$-thic $\Theta$-monoids)

${}^{\dagger}\mathcal{HT}^{\Theta} \rightsquigarrow {}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}} \rightsquigarrow \mathcal{M}^{\Theta}_{*} = \mathcal{M}^{\Theta}_{*}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}})$

$(\{{}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}}\}_{\underline{v}\in\underline{\mathbb{V}}}, {}^{\dagger}\mathcal{F}^{\Vdash}_{\mathrm{mod}})$

cat. thic

$\Pi_{\underline{X}}(\mathcal{M}^{\Theta}_{*}) \curvearrowright \underline{\underline{\mathcal{F}}}_{{}^{\dagger}\mathcal{F}^{\Theta}_{\underline{v}}} = \{\underline{\underline{\mathcal{F}}}_{{}^{\dagger}\mathcal{F}^{\Theta}_{\underline{v}}, \alpha}\}_{\alpha \in \Pi_{\underline{X}}(\mathcal{M}^{\Theta}_{*})}$ (splitting up to tor.)

(i) Kummer + cycl. rig.

$(l\Delta_{\Theta})\otimes\mathbb{Q}/\mathbb{Z} \cong \mu_N(S)$, $\underline{\underline{\mathcal{F}}}_{{}^{\dagger}\mathcal{C}_{\underline{v}}}$ ← Frob. thic mono-theta cycl. rig.

$\underline{\underline{\mathcal{F}}}_{{}^{\dagger}\mathcal{F}^{\Theta}_{\underline{v}}, \mathrm{id}} \xrightarrow[\text{Kum}]{\sim} \underline{\underline{\mathcal{F}}}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*})$ $\curvearrowright \Pi_{\underline{X}}(\mathcal{M}^{\Theta}_{*})$

(Frob)

$(\mathrm{et})$ $\Pi_{\underline{v}} \to \mathrm{Aut}_{\mathcal{D}_{\underline{v}}}(\underline{\underline{\mathcal{F}}}_{\underline{v}})$, $\mathrm{Out}^{*}\Delta^{\mathrm{cyc}}_{\underline{X}}(\Pi_{\underline{X}}(\mathcal{M}))$ compat w/ cycl. up to tor.

$\rightsquigarrow \underline{\underline{\mathcal{F}}}_{{}^{\dagger}\mathcal{F}^{\Theta}_{\underline{v}}} \xrightarrow[\text{Kum}]{\sim} \underline{\underline{\mathcal{F}}}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*})$

(ii) Kummer + cycl. rig. (classical cycl. rig.)

$\underline{\underline{\mathcal{F}}}_{{}^{\dagger}\mathcal{C}_{\underline{v}}} \xrightarrow[\text{Kum}]{\sim} \underline{\underline{\mathcal{F}}}_{\mathrm{cns}}(\mathcal{M}^{\Theta}_{*})$

const. monoids $\curvearrowleft \Pi_{\underline{X}}(\mathcal{M}^{\Theta}_{*})$

[IUTch II, Prop. 3.4] ($\Pi$-thic $\Theta$-monoids)

${}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}}$ full poly isom $\mathcal{M}^{\Theta}_{*}(\Pi_{\underline{v}}) \xrightarrow{\sim} \mathcal{M}^{\Theta}_{*}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}})$

(i) (split $\Theta$-monoids multiradial)

$\underline{\underline{\mathcal{F}}}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*}(\Pi_{\underline{v}})) \xrightarrow{\sim} \underline{\underline{\mathcal{F}}}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}})) \xrightarrow[(\text{Kum})^{-1}]{\sim} \underline{\underline{\mathcal{F}}}_{{}^{\dagger}\mathcal{F}^{\Theta}_{\underline{v}}}$

$(\Pi_{\underline{v}} \cong) \Pi_{\underline{X}}(\mathcal{M}^{\Theta}_{*}(\Pi_{\underline{v}})) \xrightarrow[\Theta]{\sim} \Pi_{\underline{X}}(\mathcal{M}^{\Theta}_{*}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}}))$

compat w/ splitting up to tor.

$\underline{\underline{\mathcal{F}}}_{\mathrm{env}}(\ldots)^{\times} \xrightarrow[\Theta]{\sim} \underline{\underline{\mathcal{F}}}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}}))^{\times} \xrightarrow[(\text{Kum})^{-1}]{\sim} \underline{\underline{\mathcal{F}}}^{\times}_{{}^{\dagger}\mathcal{F}^{\Theta}_{\underline{v}}}$

$\ldots \xrightarrow[\Theta]{\sim} G_{\underline{v}}(\mathcal{M}^{\Theta}_{*}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\underline{v}}))$

$\Pi_{\underline{v}} \ldots (\underline{\underline{\mathcal{F}}}_{\mathrm{env}} \ldots$

$$\mathcal{F}_{\mathrm{env}}(\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}}))^{\times} \xrightarrow[\circledast]{\sim} \mathcal{F}_{\mathrm{env}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}}))^{\times} \xrightarrow[(\kappa_{\mathrm{mod}})^{-1}]{\sim} \mathcal{F}^{\times}_{{}^{\dagger}\mathcal{F}^{\circledast}_{\underline{v}}}$$

$$G_{\underline{v}} \cong G_{\underline{v}}(\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}})) \xrightarrow[\circledast]{\sim} G_{\underline{v}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}}))$$

$$\Pi_{\underline{v}} \longmapsto \mathcal{F}_{\mathrm{env}}(\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}}))$$

f.r. alg'm (circledast)

compat w/ ${}^{\forall}$autom. of the pair

mult. rad.

$$G_{\underline{v}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}})) \curvearrowright (\mathcal{F}_{{}^{\dagger}\mathcal{F}^{\circledast}_{\underline{v}}})^{\times\mu}$$

Ismot-multiple of

$$G_{\underline{v}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}})) \curvearrowright \mathcal{F}^{\times}_{{}^{\dagger}\mathcal{F}^{\circledast}_{\underline{v}}}$$

(ii) (const monoid unimodiality)

each isom $\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}}) \cong \mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}})$ $\circledast$

induce

$$\mathcal{F}_{\mathrm{cns}}(\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}})) \xrightarrow{\overset{\circledast}{\sim}} \mathcal{F}_{\mathrm{cns}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}})) \xrightarrow[(\kappa_{\mathrm{mod}})^{-1}]{\sim} \mathcal{F}_{{}^{\dagger}\mathcal{F}_{\underline{v}}}$$

$$\Pi_{\underline{v}} \cong \Pi_{\underline{v}}(\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}})) \xrightarrow{\overset{\circledast}{\sim}} \Pi_{\underline{v}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}}))$$

$$\mathcal{F}_{\mathrm{cns}}(\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}}))^{\times} \xrightarrow{\overset{\circledast}{\sim}} \mathcal{F}_{\mathrm{cns}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}}))^{\times} \xrightarrow[(\kappa_{\mathrm{mod}})^{-1}]{\sim} \mathcal{F}^{\times}_{{}^{\dagger}\mathcal{F}_{\underline{v}}}$$

$$G_{\underline{v}} \cong G_{\underline{v}}(\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}})) \xrightarrow{\sim} G_{\underline{v}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}}))$$

f.r. alg'm

$$\Pi_{\underline{v}} \longmapsto \mathcal{F}_{\mathrm{cns}}(\mathcal{M}^{\circledast}_{*}(\Pi_{\underline{v}}))$$

uses Kummer rings!

autom. of the pair

not compat w/

$$G_{\underline{v}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}})) \curvearrowright (\mathcal{F}_{{}^{\dagger}\mathcal{F}_{\underline{v}}})^{\times\mu}$$

$$G_{\underline{v}}(\mathcal{M}^{\circledast}_{*}({}^{\dagger}\mathcal{D}_{\underline{v}})) \curvearrowright \mathcal{F}_{{}^{\dagger}\mathcal{F}_{\underline{v}}}$$

...ussian monoids $\Pi \to \mathcal{I}_{\mathcal{M}} \leftarrow \mathcal{F}$

[IUTch II, Cor 3.5] ($\mathcal{M}$ the Gaussian monoids)

$\mathcal{M}^{\circledast}_{*}$ s.t. $\Pi_{\underline{v}}(\mathcal{M}^{\circledast}_{*}) \cong \Pi_{\underline{v}}$

$A \in \mathrm{LabCusp}^{\pm}(\Pi_{\underline{v}}(\mathcal{M}^{\circledast}_{*}))$ ($I^{\ell}_{*} \subset \Pi_{\underline{v}}(\mathcal{M}^{\circledast}_{*})$)

(i) $A \mapsto I_{A} \subset \Pi_{\underline{v}}(\mathcal{M}^{\circledast}_{*})$

$\mathbb{F}^{\times\pm}_{\ell} = \Delta_{C}(\mathcal{M}^{\circledast}_{*}) / \Delta_{\underline{v}}(\mathcal{M}^{\circledast}_{*}) \curvearrowright \Pi_{\underline{v}}(\mathcal{M}^{\circledast}_{*})$

$\Delta_{\underline{v}}(\mathcal{M}^{\circledast}_{*})$-outer action

induces isom's of pair ($G_{\underline{v}}(\mathcal{M}^{\circledast}_{*})_{*} \curvearrowright \mathcal{F}_{\mathrm{cns}}(\mathcal{M}^{\circledast}_{*})_{*}$)

$\mathbb{F}^{\times\pm}_{\ell}$-symmetrising isom

Gaussian monoids $\Pi \twoheadrightarrow I_{\mathbb{M}} \curvearrowleft \mathcal{T}$

[IUTch II, Cor 3.5] ($\mathbb{M}$ the Gaussian monoids)

$\mathbb{M}^{\Theta}_{*}$ s.t. $\Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*}) \cong \Pi_{\underline{v}}$

$A \in \mathrm{LabCusp}^{\pm}(\Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*}))$

(i) $A \rightsquigarrow I_{A} \subset \Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*})$ $\quad (I^{\ell}_{*} \subset \Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*}))$

$\mathbb{F}_{\ell}^{\rtimes\pm} = \Delta_{C}(\mathbb{M}^{\Theta}_{*}) / \Delta_{\underline{v}}(\mathbb{M}^{\Theta}_{*}) \curvearrowright \Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*})$

$\Delta_{\underline{v}}(\mathbb{M}^{\Theta}_{*})$-outer action

} induces

isom's of pair $\left( G_{\underline{v}}(\mathbb{M}^{\Theta}_{*\circledcirc})_{*} \curvearrowright \Psi_{\mathrm{cns}}(\mathbb{M}^{\Theta}_{*})_{*} \right)$

$\underline{\mathbb{F}_{\ell}^{\rtimes\pm}\text{-symmetrising isom}}$

($\Delta = \{0, \langle > \}$)

$*, -*$ $\xrightarrow{\text{synm.}}$ identify $|*|$

(diag) $\subset \prod(\cdot)$ $\quad |*| \in |\mathbb{F}_{\ell}|$, $\mathbb{F}_{\ell}^{*}$

write $\langle |\mathbb{F}_{\ell}| \rangle$, $\langle \mathbb{F}_{\ell}^{*} \rangle$

can construct

(synchronizing indet's)

$\Psi_{\mathrm{cns}}(\mathbb{M}^{\Theta}_{*})$

with. to $\Pi_{\mathbb{M}^{\Theta}_{*\circledcirc}}$, $D^{\delta}_{*, \mu}$

$\rightsquigarrow (\Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*}) \supset) \Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*\circledcirc}) \twoheadrightarrow G_{\underline{v}}(\mathbb{M}^{\Theta}_{*\circledcirc})_{\langle |\mathbb{F}_{\ell}| \rangle}$

$\Psi_{\mathrm{cns}}(\mathbb{M}^{\Theta}_{*}) \xrightarrow{\sim} \Psi_{\mathrm{cns}}(\mathbb{M}^{\Theta}_{*})_{\langle |\mathbb{F}_{\ell}| \rangle}$

compat w/ symm. isoms, well-def'd up to $\Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*})$-inner

indep. of $|*| \in |\mathbb{F}_{\ell}|$

conj. synchro

(ii) (Gaussian monoids)

$\underline{\underline{\Theta}}^{\mathbb{F}_{\ell}^{*}}_{\mathrm{env}}(\mathbb{M}^{\Theta}_{*\circledcirc}) := \prod_{|*| \in \mathbb{F}_{\ell}^{*}} \underline{\underline{\Theta}}^{|*|}_{\mathrm{env}}(\mathbb{M}^{\Theta}_{*\circledcirc}) \subseteq \prod_{|*| \in \mathbb{F}_{\ell}^{*}} \Psi_{\mathrm{cns}}(\mathbb{M}^{\Theta}_{*})_{|*|}$

$\# = (2\ell)^{\ell^{*}}$

call any elt value-profile

" $\{ \underline{\underline{q}}^{j^2}_{v} \}_{j \in \mathbb{F}_{\ell}^{*}}$ "

mp to $\mathbb{M}_{2\ell}$

symm. isom's in (i) & Cor 2.8 (ii)

(restriction) $\underline{\underline{\Theta}}^{?}_{\mathrm{env}}(\mathbb{M}^{\Theta}_{*})$)

$\mathbb{M}^{\Theta}_{*} \longmapsto (\infty)\Psi_{\mathrm{gau}}(\mathbb{M}^{\Theta}_{*}) := \{ (\infty)\Psi_{\xi}(\mathbb{M}^{\Theta}_{*}) := \Psi^{\times}_{\mathrm{cns}}(\mathbb{M}^{\Theta}_{*})_{\langle \mathbb{F}_{\ell}^{*} \rangle} \cdot \xi^{\mathbb{N}} \}$

$\xi$: val. profile

$\underline{\text{Gaussian monoid}}$

$G_{\underline{v}}(\mathbb{M}^{\Theta}_{*\circledcirc})_{\langle \mathbb{F}_{\ell}^{*} \rangle} \curvearrowright$

$\Psi^{\times}_{\mathrm{cns}}(\mathbb{M}^{\Theta}_{*})_{\langle \mathbb{F}_{\ell}^{*} \rangle} \subset \prod_{|*| \in \mathbb{F}_{\ell}^{*}} \Psi_{\mathrm{cns}}(\mathbb{M}^{\Theta}_{*})_{|*|}$

$(\mathbb{Q}_{\geq 0})$

$(\infty)\Psi^{?}_{\mathrm{env}}(\mathbb{M}^{\Theta}_{*}) \longrightarrow$ ... $(\mathbb{M}^{\Theta}_{*})$

$\Pi_{\underline{v}}(\mathbb{M}^{\Theta}_{*}) \curvearrowright$

(ii) (Gaussian monoids)

$\rightarrow \underline{\Theta}^{\mathbb{F}_\ell^{*}}_{env}(\mathbb{M}^{\Theta}_{*\circledast}) := \prod_{|t| \in \mathbb{F}_\ell^{*}} \underline{\Theta}^{|t|}_{env}(\mathbb{M}^{\Theta}_{*\circledast}) \subseteq \prod_{|t| \in \mathbb{F}_\ell^{*}} \mathcal{F}_{cns}(\mathbb{M}^{\Theta}_{*})_{|t|}$

$\# = (2\ell)^{\ell^*}$

call ang. elt **value-profile**

$\{ \underline{q}^{j^2} \}_{j \in \mathbb{F}_\ell^{*}}$

map to $\mathbb{M}_{2\ell}$

symm. isom's in (i) & Cn 2.8 (ii) (restrictions of $\underline{\Theta}^{?}_{env}(\mathbb{M}^{\Theta}_{*})$)

$\mathbb{M}^{\Theta}_{*} \longmapsto (\infty)\mathcal{F}_{gau}(\mathbb{M}^{\Theta}_{*}) := \{ (\infty)\mathcal{F}_{\xi}(\mathbb{M}^{\Theta}_{*}) := \mathcal{F}^{\times}_{cns}(\mathbb{M}^{\Theta}_{*})_{\langle \mathbb{F}_\ell^{*} \rangle} \}$ $\xi$: val. profile

$G_v(\mathbb{M}^{\Theta}_{*\circledast})_{\langle \mathbb{F}_\ell^{*} \rangle}$ $(\Theta_{\mathbb{Z}\geq 0})^{N}$

$\prod_{|t| \in \mathbb{F}_\ell^{*}} \mathcal{F}_{cns}(\mathbb{M}^{\Theta}_{*})_{|t|}$

**Gaussian monoid**

$\subset \Pi$

$(\infty)\mathcal{F}^{?}_{env}(\mathbb{M}^{\Theta}_{*}) \xrightarrow{\sim} \mathcal{F}_{\xi}(\mathbb{M}^{\Theta}_{*})$

$(\Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*}) \supset) \Pi_{\underline{\times}\mu}(\mathbb{M}^{\Theta}_{*\circledast})$ $\{ G_v(\mathbb{M}^{\Theta}_{*\circledast})_{|t|} \}_{|t| \in \mathbb{F}_\ell^{*}}$

$\Pi_{\underline{\times}\mu}(\mathbb{M}^{\Theta}_{*\circledast}) \curvearrowright G_v(\mathbb{M}^{\Theta}_{*})$

---

…oids $\Pi \quad \mathcal{I}_{\mathbb{M}} \quad \mathcal{F}$

…h II, Cn 3.5] ($\mathbb{M}$ tht'c Gaussian monoids)

$\mathbb{M}^{\Theta}_{*}$ s.t. $\Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*}) \cong \Pi_{\underline{\times}}$

$\pm(\Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*}))$ $(\mathcal{I}^{\ell}_{*} \subset \Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*}))$

$A \in \mathrm{LabCusp}$

$\ast \curvearrowright \mathcal{I}_{A} \subset \Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*})$

$\Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*}) \curvearrowright \Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*})$

$\Delta_{\underline{\times}}(\mathbb{M}^{\Theta}_{*})$-outer action

$\mathbb{F}_\ell^{\rtimes\pm} = \Delta_C(\mathbb{M}^{\Theta}_{*}) / \Delta_{\underline{\times}}(\mathbb{M}^{\Theta}_{*})$

$\Delta_{\underline{\times}}(\mathbb{M}^{\Theta}_{*}) \curvearrowright \mathcal{F}_{cns}(\mathbb{M}^{\Theta}_{*})_{*}$

…dnces $(G_v(\mathbb{M}^{\Theta}_{*\circledast})_{*}$

$\Delta = \{0, \langle\, \rangle\}$

$t, -t$ symm. identify $|t|$

$(\mathrm{diag}) \subset \prod_{|t| \in |\mathbb{F}_\ell|} \mathbb{F}_\ell^{*}$

write $\langle |\mathbb{F}_\ell| \rangle$ $\langle \mathbb{F}_\ell^{*} \rangle$

can construct (synchronizing indet's)

$\mathbb{M}, \mathcal{F}, \Pi$

$\mathcal{F}_{cns}(\mathbb{M}^{\Theta}_{*})$ $D^{\delta}_{t,\mu}$

restr. to $\Pi_{\underline{\times}\mu}(\mathbb{M}^{\Theta}_{*\circledast}) \hookrightarrow \Pi_{\underline{\times}\mu}(\mathbb{M}^{\Theta}_{*\circledast}) \curvearrowright G_v(\mathbb{M}^{\Theta}_{*\circledast})_{\langle|\mathbb{F}_\ell|\rangle}$

$\sim (\Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*}) \curvearrowright \Pi_{\underline{\times}\mu}(\mathbb{M}^{\Theta}_{*\circledast}))$ $\mathcal{F}_{cns}(\mathbb{M}^{\Theta}_{*})_{\langle|\mathbb{F}_\ell|\rangle}$

$\mathcal{F}_{cns}(\mathbb{M}^{\Theta}_{*}) \xrightarrow{\sim} \mathcal{F}_{cns}(\mathbb{M}^{\Theta}_{*})_{\langle|\mathbb{F}_\ell|\rangle}$ indep.

compat. w/ symm. isoms, well-def. up to $\Pi_{\underline{\times}}(\mathbb{M}^{\Theta}_{*})$-indet.

indep. of $|t| \in |\mathbb{F}_\ell|$ conj. synchro collection of $\overline{\mathcal{F}}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*}) \xrightarrow{\sim} \overline{\mathcal{F}}_{\mathrm{gau}}(\mathcal{M}^{\Theta}_{*})$

$\mathcal{O}_{\geq 0}$ end (as)

(iii) (const. monoids & splittings)

$$\mathcal{O}^{\times}_{F_{\mathrm{mod}}} \times \underline{\underline{\Theta}}^{\mathbb{N}} \xrightarrow{\sim} \mathcal{O}_{F_{\mathrm{mod}}} \{ \underline{\underline{q}}^{j^2} \}^{\mathbb{N}}$$

$0 \in |\mathbb{F}_l|$

$$\overline{\mathcal{F}}_{\mathrm{cns}}(\mathcal{M}^{\Theta}_{*})_{\langle |\mathbb{F}_l| \rangle} \quad \underset{\text{graph}}{\text{restr}} \quad \overline{\mathcal{F}}_{\mathrm{cns}}(\mathcal{M}^{\Theta}_{*})|_{0} \xrightarrow{\sim} \overline{\mathcal{F}}_{\mathrm{cns}}(\mathcal{M}^{\Theta}_{*})_{\langle \mathbb{F}_l^{*} \rangle}$$

zero

compat w/ $G_{v}(\mathcal{M}^{\Theta}_{*})$ each label

$(\mathbb{Q}_{>0})$

restr. to 0-label

$\rightsquigarrow$ splitting up to tor of Gaussian monoid

$$\overline{\mathcal{F}}_{\mathrm{gau}}(\mathcal{M}^{\Theta}_{*}) = \overline{\mathcal{F}}^{\times}_{\mathrm{cns}}(\mathcal{M}^{\Theta}_{*})_{\langle \mathbb{F}_l^{*} \rangle} \{ \; \}^{(\mathcal{O}_{\geq 0})^{\mathbb{N}}}$$

(iv)

[IUTch II, Cor 3.6] ($\mathcal{F}$-thic Gaussian monoids)

${}^{\dagger}\underline{\underline{\mathcal{F}}}_{\succ} \rightsquigarrow \mathbb{M}_{*}^{\circledast} = \mathbb{M}_{*}^{\circledast}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\succ})$

(i) $\underline{\Psi}_{{}^{\dagger}\mathcal{C}_{\succ}} \xrightarrow[\text{Kmm}]{\sim} \underline{\Psi}_{\mathrm{cnst}}(\mathbb{M}_{*}^{\circledast})$

(Frol)

$\rightsquigarrow$ $\mathrm{Aut}_{\mathrm{ell}}(\mathrm{Cusp}^{\pm}(\Pi_{\underline{X}}(\mathbb{M}_{*}^{\circledast})))$

(et)

$(\underline{\Psi}_{{}^{\dagger}\mathcal{C}_{\succ}})_{A} \xrightarrow[]{\sim\ \text{Kmm}} \underline{\Psi}_{\mathrm{cns}}(\mathbb{M}_{*}^{\circledast})_{A}$

$\cap \qquad \cap$

$G_{\succ}(\mathbb{M}_{*}^{\circledast})_{A} \xrightarrow{\sim} G_{\succ}(\mathbb{M}_{*\circledast}^{\circledast})_{A}$

gau.

$\mathbb{F}_{l}^{\times\pm}$-symm. isom's

$\wr\wr$

$\Delta_{c}(\mathbb{M}_{*}^{\circledast}) / \Delta_{\underline{X}}(\mathbb{M}_{*}^{\circledast})$

well-def up to $\Pi_{\underline{X}}(\mathbb{M}_{*}^{\circledast})$-inner

indep. of $*$

$\xi$: mono-profile

by (Kmm)

(Frol)

${}^{(\infty)}\underline{\Psi}_{\mathcal{F}_{\xi}}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\succ}) \subseteq \prod_{|t| \in \mathbb{F}_{l}^{*}} (\underline{\Psi}_{{}^{\dagger}\mathcal{C}_{\succ}})_{|t|}$

${}^{(\infty)}\underline{\Psi}_{\mathcal{F}_{\mathrm{gau}}} := \{{}^{(\infty)}\underline{\Psi}_{\mathcal{F}_{\xi}}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\succ})\}_{\xi}$

$\cap$

$G_{\succ}(\mathbb{M}_{*}^{\circledast})_{\langle \mathbb{F}_{l}^{*} \rangle}$

(Frol) (et) (et) (Frol)

${}^{(\infty)}\underline{\Psi}_{\mathcal{F}_{\succ, \mathrm{id}}^{\dagger\circledast}} \xrightarrow[\text{Kmm}]{\sim} {}^{(\infty)}\underline{\Psi}_{\mathrm{env}}^{l}(\mathbb{M}_{*}^{\circledast}) \xrightarrow[\sim]{\text{emb}} {}^{(\infty)}\underline{\Psi}_{\xi}(\mathbb{M}_{*}^{\circledast}) \xrightarrow[\text{(Kmm)}]{\sim} {}^{(\infty)}\underline{\Psi}_{\mathcal{F}_{\xi}}({}^{\dagger}\underline{\underline{\mathcal{F}}}_{\succ})$

$\Pi_{\underline{X}}(\mathbb{M}_{*}^{\circledast}) \curvearrowright \Pi_{\circledast}(\mathbb{M}_{*\circledast}^{\circledast})$

$\{G_{\succ}(\mathbb{M}_{*\circledast}^{\circledast})_{|t|}\}_{|t| \in \mathbb{F}_{l}^{*}}$

$\xrightarrow{\sim} \{G_{\succ}(\mathbb{M}_{*}^{\circledast})_{|t|}\}_{|t| \in \mathbb{F}_{l}^{*}}$

compat as above

$(\infty)\ \mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\triangle} \xrightarrow[(\mathrm{Kmm})^{(\infty)}]{\sim} \mathcal{F}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*}) \underset{\mathrm{anal}}{\cong} \mathcal{F}_{\mathrm{gau}}(\mathcal{M}^{\Theta}_{*}) \underset{(\mathrm{Kmm})^{-1}}{\cong} \mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\mathrm{gau}}({}^{\dagger}\mathcal{F}_{\triangle})$

(Frob) (et) (et) (Frob)

(iii) (const monoids & splittings)

$0 \in |\mathbb{F}_l| \rightsquigarrow (\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\triangle})_{0} \cong (\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\triangle})_{\langle \mathbb{F}_l^{*} \rangle}$

splitting ${}_{(\infty)}\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\mathcal{F}_\xi}({}^{\dagger}\mathcal{F}_{\triangle}) = (\mathcal{F}^{\Vdash\blacktriangleright\times\mu}_{\triangle})_{\langle \mathbb{F}_l^{*} \rangle} \cdot \mathrm{Im}(\xi)^{\wedge}$ $\mathcal{O}_{\geq 0}$

$\Pi$

next, cusic ${}^{\circ}\Pi_{\triangle}$

later use for

full poly

$\mathcal{M}^{\Theta}_{*}(\Pi_{\triangle}) \cong \mathcal{M}^{\Theta}_{*}({}^{\dagger}\mathcal{F}_{\triangle})$

each isom

[IUTch II, Cor 3.7] $\circledast$

$_{(\infty)}\mathcal{F}_{\mathrm{env}}(\mathcal{M}^{\Theta}_{*}(\Pi_{\triangle})) \underset{\mathrm{anal}}{\cong} {}_{(\infty)}\mathcal{F}_{\xi}(\mathcal{M}^{\Theta}_{*}(\Pi_{\triangle})) \underset{\circledast}{\cong} {}_{(\infty)}\mathcal{F}_{\xi}(\mathcal{M}^{\Theta}_{*}({}^{\dagger}\mathcal{F}_{\triangle})) \xrightarrow[(\mathrm{Kmm})^{-1}]{\sim} {}_{(\infty)}\mathcal{F}_{\mathcal{F}_\xi}({}^{\dagger}\mathcal{F}_{\triangle})$

Gaussian monoid mono-analyticity